НЕОДРЕЂЕНИ ИНТЕГРАЛИ

Метод смене

1. $$\boxed{\int e^{3x-4}dx}\ \left|\begin{matrix} 3x-4=t \\ 3dx=dt \\ dx=\dfrac{dt}{3} \end{matrix}\right| = \int e^t\cdot\frac{dt}{3} = \frac{1}{3}\int e^t dt = \frac{1}{3}e^t = \underline{\underline{\frac{1}{3}e^{3x-4}+C}}$$

2. $$\boxed{\int e^{\sin x}\cdot\cos x\,dx}\ \left|\begin{matrix} \sin x=t \\ \cos x\,dx=dt \\ dx=\dfrac{dt}{\cos x} \end{matrix}\right| = \int e^t\cdot\cos x\cdot\frac{dt}{\cos x} = \int e^t dt = e^t = \underline{\underline{e^{\sin x}+C}}$$

3. $$\boxed{\int x^3\cdot e^{x^4}dx}\ \left|\begin{matrix} x^4=t \\ 4x^3dx=dt \\ dx=\dfrac{dt}{4x^3} \end{matrix}\right| = \int x^3\cdot e^t\cdot\frac{dt}{4x^3} = \frac{1}{4}\int e^t dt = \frac{1}{4}e^t = \underline{\underline{\frac{1}{4}e^{x^4}+C}}$$

4. $$\boxed{\int \sin^2 x\cdot\cos x\,dx}\ \left|\begin{matrix} \sin x=t \\ \cos x\,dx=dt \\ dx=\dfrac{dt}{\cos x} \end{matrix}\right| = \int t^2\cdot\cos x\cdot\frac{dt}{\cos x} = \int t^2 dt = \frac{t^3}{3} = \underline{\underline{\frac{(\sin x)^3}{3}+C}}$$

5. $$\boxed{\int \frac{\ln x}{x}dx}\ \left|\begin{matrix} \ln x=t \\ \dfrac{1}{x}dx=dt \\ dx=xdt \end{matrix}\right| = \int \frac{t}{x}\cdot xdt = \int tdt = \frac{t^2}{2} = \underline{\underline{\frac{(\ln x)^2}{2}+C}}$$

6. $$\boxed{\int \frac{1}{x\cdot\ln^2 x}dx}\ \left|\begin{matrix} \ln x=t \\ \dfrac{1}{x}dx=dt \\ dx=xdt \end{matrix}\right| = \int \frac{1}{x\cdot t^2}\cdot xdt = \int \frac{1}{t^2}dt = -\frac{1}{t} = \underline{\underline{-\frac{1}{\ln x}+C}}$$

7. $$\boxed{\int \frac{1}{x\cdot\ln x}dx}\ \left|\begin{matrix} \ln x=t \\ \dfrac{1}{x}dx=dt \\ dx=xdt \end{matrix}\right| = \int \frac{1}{x\cdot t}xdt = \int \frac{1}{t}dt = \ln|t| = \underline{\underline{\ln|\ln x|+C}}$$

8. $$\boxed{\int e^x\cdot\sqrt{2+3e^x}dx}\ \left|\begin{matrix} 2+3e^x=t^2 \\ t=\sqrt{2+3e^x} \\ 3e^xdx=2tdt \\ dx=\dfrac{2tdt}{3e^x} \end{matrix}\right| = \int e^x\cdot\sqrt{t^2}\cdot\frac{2tdt}{3e^x} = \frac{2}{3}\int t^2dt = \frac{2}{3}\cdot\frac{t^3}{3} = \underline{\underline{\frac{2}{9}\left(\sqrt{2+3e^x}\right)^3+C}}$$

9. $$\boxed{\int \frac{\cos x}{\sqrt{1+\sin x}}dx}\ \left|\begin{matrix} 1+\sin x=t^2 \\ t=\sqrt{1+\sin x} \\ \cos x\,dx=2tdt \\ dx=\dfrac{2tdt}{\cos x} \end{matrix}\right| = \int \frac{\cos x}{\sqrt{t^2}}\cdot\frac{2tdt}{\cos x} = 2\int dt = 2\cdot t = \underline{\underline{2\sqrt{1+\sin x}+C}}$$

10. $$\boxed{\int \frac{\cos x}{\sqrt{3+\sin x}}dx}\ \left|\begin{matrix} 3+\sin x=t^2 \\ t=\sqrt{3+\sin x} \\ \cos x\,dx=2tdt \\ dx=\dfrac{2tdt}{\cos x} \end{matrix}\right| = \int \frac{\cos x}{\sqrt{t^2}}\cdot\frac{2tdt}{\cos x} = 2\int dt = 2\cdot t = \underline{\underline{2\sqrt{3+\sin x}+C}}$$

11. $\boxed{\int \frac{\sin x}{\sqrt{1+2\cos x}}dx}$ $\left| \begin{array}{c} 1+2\cos x = t^2 \\ t=\sqrt{1+2\cos x} \\ -2\sin x\,dx = 2tdt \\ dx = \frac{2tdt}{-2\sin x} \\ dx = -\frac{tdt}{\sin x} \end{array} \right| = \int \frac{\sin x}{\sqrt{t^2}} \cdot \frac{-tdt}{\sin x} = -\int dt = -t = \underline{\underline{-\sqrt{1+2\cos x}+C}}$

12. $\boxed{\int \frac{\sin x}{\cos^2 x}dx}$ $\left| \begin{array}{c} \cos x = t \\ -\sin x\,dx = dt \\ dx = \frac{dt}{-\sin x} \end{array} \right| = \int \frac{\sin x}{t^2} \cdot \frac{dt}{-\sin x} = -\int \frac{1}{t^2}dt = -\left(-\frac{1}{t}\right) = \frac{1}{t} = \underline{\underline{\frac{1}{\cos x}+C}}$

13. $\boxed{\int \frac{\cos(\ln x)}{x}dx}$ $\left| \begin{array}{l} \ln x = t \\ \frac{1}{x}dx = dt \\ dx = xdt \end{array} \right| = \int \frac{\cos|t|}{x} \cdot xdt = \int \cos|t|\,dt = \sin|t| = \underline{\underline{\sin|\ln x|+C}}$

14. $\boxed{\int tg\,x\,dx} = \boxed{\int \frac{\sin x}{\cos x}dx}$ $\left| \begin{array}{c} \cos x = t \\ -\sin x\,dx = dt \\ dx = \frac{dt}{-\sin x} \end{array} \right| = \int \frac{\sin x}{t} \cdot \frac{dt}{-\sin x} = -\int \frac{1}{t}dt = -\ln|t| = \underline{\underline{-\ln|\cos x|+C}}$

15. $\boxed{\int ctg\,x\,dx} = \boxed{\int \frac{\cos x}{\sin x}dx}$ $\left| \begin{array}{c} \sin x = t \\ \cos x\,dx = dt \\ dx = \frac{dt}{\cos x} \end{array} \right| = \int \frac{\cos x}{t} \cdot \frac{dt}{\cos x} = \int \frac{1}{t}dt = \ln|t| = \underline{\underline{\ln|\sin x|+C}}$

16. $\boxed{\int \frac{\ln x - 3}{x \cdot \sqrt{\ln x}}dx}$ $\left| \begin{array}{l} \ln x = t^2 \\ t = \sqrt{\ln x} \\ \frac{1}{x}dx = 2tdt \\ dx = 2xtdt \end{array} \right| = \int \frac{t^2-3}{x\cdot\sqrt{t^2}} \cdot 2xtdt = 2\int (t^2-3)dt = 2\int t^2dt - 2\int 3dt = 2\left(\frac{t^3}{3} - 3\int dt\right) = 2\left(\frac{t^3}{3} - 3t\right) = \underline{\underline{2\left(\frac{\sqrt{\ln x}^3}{3} - 3\sqrt{\ln x}\right)+C}}$

17. $\boxed{\int \frac{x}{\sqrt{3x^2-1}}dx}$ $\left| \begin{array}{c} 3x^2-1 = t^2 \\ t = \sqrt{3x^2-1} \\ 6xdx = 2tdt \\ dx = \frac{2tdt}{6x} \\ dx = \frac{tdt}{3x} \end{array} \right| = \int \frac{x}{\sqrt{t^2}} \cdot \frac{tdt}{3x} = \frac{1}{3}\int dt = \frac{1}{3}t = \underline{\underline{\frac{1}{3}\sqrt{3x^2-1}+C}}$

18. $\boxed{\int \frac{1}{9-13x}dx}$ $\left| \begin{array}{l} 9-13x = t \\ -13dx = dt \\ dx = \frac{dt}{-13} \end{array} \right| = \int \frac{1}{t} \cdot \frac{dt}{-13} = -\frac{1}{13}\int \frac{1}{t}dt = -\frac{1}{13}\ln|t| = \underline{\underline{-\frac{1}{13}\ln|9-13x|+C}}$

19. $\boxed{\int \frac{1}{1+e^{-x}}dx} = \int \frac{1}{1+\frac{1}{e^x}}dx = \int \frac{1}{\frac{e^x+1}{e^x}}dx = \int \frac{e^x}{e^x+1}dx$

$\left| \begin{array}{l} e^x+1 = t \\ e^x dx = dt \\ dx = \frac{dt}{e^x} \end{array} \right| = \int \frac{e^x}{t} \cdot \frac{dt}{e^x} = \int \frac{1}{t}dt = \ln|t| = \underline{\underline{\ln|e^x+1|+C}}$

ТИП I

20.

$$\boxed{\int \frac{1}{-x^2+3x-2}dx} = -\int \frac{1}{x^2-3x+2}dx = -\int \frac{1}{x^2-3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+2}dx$$

$$= -\int \frac{1}{\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+2}dx = -\int \frac{1}{\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+\frac{8}{4}}dx = -\int \frac{1}{\left(x-\frac{3}{2}\right)^2-\frac{1}{4}}dx$$

$$\left|\begin{matrix} x-\frac{3}{2}=t \\ dx=dt \end{matrix}\right| = -\int \frac{1}{t^2-\left(\frac{1}{2}\right)^2}dx = -\frac{1}{2\cdot\frac{1}{2}}\ln\frac{t-\frac{1}{2}}{t+\frac{1}{2}} = -\ln\left|\frac{x-\frac{3}{2}-\frac{1}{2}}{x-\frac{3}{2}+\frac{1}{2}}\right| = \underline{\underline{-\ln\left|\frac{x-2}{x-1}\right|+C}}$$

21.

$$\boxed{\int \frac{1}{2x^2-12x+26}dx} = \frac{1}{2}\int \frac{1}{x^2-6x+13}dx = \frac{1}{2}\int \frac{1}{x^2-6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+13}dx$$

$$= \frac{1}{2}\int \frac{1}{(x-3)^2-9+13}dx = \frac{1}{2}\int \frac{1}{(x-3)^2+4}dx$$

$$\left|\begin{matrix} x-3=t \\ dx=dt \end{matrix}\right| = \frac{1}{2}\int \frac{1}{t^2+2^2}dx = \frac{1}{2}\cdot\frac{1}{2}arctg\frac{t}{2} = \underline{\underline{\frac{1}{4}arctg\frac{x-3}{2}+C}}$$

Уради сам:

22. $\boxed{\int \frac{1}{x^2+6x+7}dx}$ ... $= \underline{\underline{\frac{1}{8}\ln\left|\frac{x-1}{x+7}\right|+C}}$

23. $\boxed{\int \frac{1}{x^2+2x+1}dx}$ ... $= \underline{\underline{-\frac{1}{x+1}+C}}$

24. $\boxed{\int \frac{1}{x^2-6x+9}dx}$ ... $= \underline{\underline{-\frac{1}{x-3}+C}}$

25. $\boxed{\int \frac{1}{x^2+2x+5}dx}$ ... $= \underline{\underline{\frac{1}{2}arctg\frac{x+1}{2}+C}}$

ТИП II

26. $\boxed{\int \frac{3x-4}{x^2-5x+4}dx}$ $\quad A=3,\ B=-4 \quad a=1,\ b=-5,\ c=4$

$$\frac{3}{2}\ln|x^2-5x+4| + \left(-4-\frac{3(-5)}{2\cdot 1}\right)\cdot\int \frac{1}{x^2-5x+4}dx$$

$$\frac{3}{2}\ln|x^2-5x+4| + \left(-4-\frac{-15}{2}\right)\cdot\int \frac{1}{x^2-5x+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+4}dx$$

$$\frac{3}{2}\ln|x^2-5x+4| + \left(-4+\frac{15}{2}\right)\cdot\int \frac{1}{\left(x-\frac{5}{2}\right)^2-\frac{9}{4}}dx \qquad \left|\begin{matrix} x-\frac{5}{2}=t \\ dx=dt \end{matrix}\right|$$

$$\frac{3}{2}\ln|x^2-5x+4| + \frac{7}{2}\cdot\int \frac{1}{t^2-\left(\frac{3}{2}\right)^2}dt = \frac{3}{2}\ln|x^2-5x+4| + \frac{7}{2}\cdot\frac{1}{2\cdot\frac{3}{2}}\ln\frac{t-\frac{3}{2}}{t+\frac{3}{2}}$$

$$\frac{3}{2}\ln|x^2-5x+4|+\frac{7}{2}\cdot\ln\frac{x-\frac{5}{2}-\frac{3}{2}}{x-\frac{5}{2}+\frac{3}{2}}=\underline{\underline{\frac{3}{2}\ln|x^2-5x+4|+\frac{7}{2}\ln\frac{x-4}{x-1}+C}}$$

27. $\boxed{\int\frac{4-3x}{x^2+2x+4}dx}$ $\quad A=-3,\ B=4 \quad a=1,\ b=2,\ c=4$

$$-\frac{3}{2}\ln|x^2+2x+4|+\left(4-\frac{-3\cdot 2}{2\cdot 1}\right)\cdot\int\frac{1}{x^2+2x+4}dx$$

$$-\frac{3}{2}\ln|x^2+2x+4|+7\cdot\int\frac{1}{x^2+2x+\left(\frac{2}{2}\right)^2-\left(\frac{2}{2}\right)^2+4}dx$$

$$-\frac{3}{2}\ln|x^2+2x+4|+7\cdot\int\frac{1}{(x+1)^2+3}dx \qquad \left|\begin{matrix}x+1=t\\ dx=dt\end{matrix}\right|$$

$$-\frac{3}{2}\ln|x^2+2x+4|+7\cdot\int\frac{1}{t^2+\sqrt{3}^2}dx=-\frac{3}{2}\ln|x^2+2x+4|+7\cdot\frac{1}{\sqrt{3}}arctg\frac{t}{\sqrt{3}}$$

$$=\underline{\underline{-\frac{3}{2}\ln|x^2+2x+4|+\frac{7}{\sqrt{3}}arctg\frac{x+1}{\sqrt{3}}+C}}$$

Уради сам:

28. $\boxed{\int\frac{5x+2}{x^2+4x+4}dx}\ \ldots=\underline{\underline{\frac{5}{2}\ln|x^2+4x+4|+\frac{8}{x+2}+C}}$

29. $\boxed{\int\frac{2x+3}{x^2-6x+12}dx}\ \ldots=\underline{\underline{\ln|x^2-6x+12|+\frac{9}{\sqrt{3}}arctg\frac{x-3}{\sqrt{3}}+C}}$

ТИП III

30. $I=\boxed{\int\frac{x^3+2}{x^3-3x^2+2x}dx}$

$$x^3+2 : x^3-3x^2+2x=1+\frac{3x^2-2x+2}{x^3-3x^2+2x}$$
$$\underline{-x^3\mp 3x^2\pm 2x}$$
$$3x^2-3x+2$$

$$I=\int 1\,dx+\int\frac{3x^2-2x+2}{x^3-3x^2+2x}dx=\underline{X+I_1}$$

$$I_1=\int\frac{3x^2-2x+2}{x^3-3x^2+2x}dx$$

$$\frac{3x^2-2x+2}{x^3-3x^2+2x}=\frac{3x^2-2x+2}{x\underbrace{(x^2-3x+2)}}=\frac{3x^2-2x+2}{x(x-2)(x-1)}$$

$$x_{1,2}=\frac{3\pm\sqrt{9-4\cdot 2}}{2}=\frac{3\pm 1}{2}=\begin{matrix}2\\1\end{matrix}$$

$$\frac{3x^2-2x+2}{x(x-2)(x-1)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x-1} \qquad /x(x-2)(x-1)$$

$$3x^2-2x+2=A(x^2-3x+2)+Bx(x-1)+Cx(x-2)$$

$$3x^2-2x+2=Ax^2-3Ax+2A+Bx^2-Bx+Cx^2-2Cx$$

$$3x^2-2x+2=x^2(A+B+C)+x(-3A-B-2C)+2A$$

$$A+B+C=3\longrightarrow \qquad B+C=2\Longrightarrow \boxed{B=5}$$
$$-3A-B-2C=-2\longrightarrow \qquad -B-2C=1$$
$$2A=2\Longrightarrow\boxed{A=1} \qquad -C=3\Longrightarrow\boxed{C=-3}$$

II начин

$\boxed{A}$ $\lim_{x\to 0}\frac{3x^2-2x+2}{(x-2)(x-1)}=\frac{2}{-2(-1)}=\boxed{1}$

$\boxed{B}$ $\lim_{x\to 1}\frac{3x^2-2x+2}{x(x-1)}=\frac{12-4+2}{2}=\boxed{5}$

$\boxed{C}$ $\lim_{x\to 2}\frac{3x^2-2x+2}{x(x-2)}=\frac{2}{-1}=\boxed{-3}$

$$\int\frac{3x^2-2x+2}{x(x-2)(x-1)}dx=\int\frac{A}{x}dx+\int\frac{B}{x-2}dx+\int\frac{C}{x-1}dx=\int\frac{1}{x}dx+\int\frac{5}{x-2}dx+\int\frac{-3}{x-1}dx$$

$I_1=\ln|x|+5\ln|x-2|-3\ln|x-1|$

$I=x+I_1$ $\qquad$ $\underline{\underline{I=x+\ln|x|+5\ln|x-2|-3\ln|x-1|+C}}$

$$x^6+13:x^4-5x^2+4=x^2+5+\frac{21x^2-7}{x^4-5x^2+4}$$

31. $I=\boxed{\int\frac{x^6+13}{x^4-5x^2+4}dx}$

$$\begin{array}{l}\underline{-x^6\mp 5x^4\pm 4x^2}\\ \quad 5x^4-4x^2+13\\ -\underline{5x^4\mp 25x^2\pm 20}\\ \quad\quad 21x^2-7\end{array}$$

$$I=\int x^2dx+\int 5dx+\int\frac{21x^2-7}{x^4-5x^2+4}dx=\frac{x^3}{3}+5\cdot x+I_1$$

$$I_1=\int\frac{21x^2-7}{x^4-5x^2+4}dx=7\cdot\int\frac{3x^2-1}{x^4-5x^2+4}dx\qquad\boxed{x^2=t}$$

$$x^4-5x^2+4=t^2-5t+4\qquad t_{1,2}=\frac{5\pm\sqrt{25-4\cdot 4}}{2}=\frac{5\pm 3}{2}=\begin{matrix}4\\1\end{matrix}$$

$$x^4-5x^2+4=t^2-5t+4=(t-1)(t-4)=(x^2-1)(x^2-4)$$
$$=(x-1)(x+1)(x-2)(x+2)$$

$$\int\frac{3x^2-1}{x^4-5x^2+4}dx=\int\frac{3x^2-1}{(x-1)(x+1)(x-2)(x+2)}dx$$

$$\frac{3x^2-1}{(x-1)(x+1)(x-2)(x+2)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x-2}+\frac{D}{x+2}$$

$\boxed{A}$ $\lim_{x\to 1}\frac{3x^2-1}{(x+1)(x-2)(x+2)}=\frac{3-1}{2(-1)\cdot 3}=\frac{2}{-6}=\boxed{-\frac{1}{3}}$

$\boxed{B}$ $\lim_{x\to -1}\frac{3x^2-1}{(x-1)(x-2)(x+2)}=\frac{3-1}{-2(-3)\cdot 1}=\frac{2}{6}=\boxed{\frac{1}{3}}$

$\boxed{C}$ $\lim_{x\to 1}\frac{3x^2-1}{(x-1)(x+1)(x+2)}=\frac{12-1}{1\cdot 3\cdot 4}=\boxed{\frac{11}{12}}$

$\boxed{D}$ $\lim_{x\to 1}\frac{3x^2-1}{(x-1)(x+1)(x-2)}=\frac{12-1}{-3\cdot(-1)\cdot(-4)}=\boxed{\frac{11}{-12}}$

$$\int\frac{3x^2-1}{(x-1)(x+1)(x-2)(x+2)}dx=\int\frac{A}{x-1}dx+\int\frac{B}{x+1}dx+\int\frac{C}{x-2}dx+\int\frac{D}{x+2}dx$$

$$\int\frac{-\frac{1}{3}}{x-1}dx+\int\frac{\frac{1}{3}}{x+1}dx+\int\frac{\frac{11}{12}}{x-2}dx+\int\frac{-\frac{11}{12}}{x+2}dx$$

$$-\frac{1}{3}\ln|x-1|+\frac{1}{3}\ln|x+1|+\frac{11}{12}\ln|x-2|-\frac{11}{12}\ln|x+2|$$

$$I=\frac{x^3}{3}+5x+7\cdot\left(-\frac{1}{3}\ln|x-1|+\frac{1}{3}\ln|x+1|+\frac{11}{12}\ln|x-2|-\frac{11}{12}\ln|x+2|\right)$$

$$I=\frac{x^3}{3}+5x-\frac{7}{3}\ln|x-1|+\frac{7}{3}\ln|x+1|+\frac{77}{12}\ln|x-2|-\frac{77}{12}\ln|x+2|+C$$

32. $\boxed{\int\frac{2x-1}{x^4-x^3}dx}$ $\qquad \frac{2x-1}{x^3(x-1)}=\frac{A}{x^3}+\frac{B}{x^2}+\frac{C}{x}+\frac{D}{x-1} \qquad /x^3(x-1)$

$$2x-1=A(x-1)+Bx(x-1)+Cx^2(x-1)+Dx^3$$
$$2x-1=Ax-A+Bx^2-Bx+Cx^3-Cx^2+Dx^3$$
$$2x-1=x^3(C+D)+x^2(B-C)+x(A-B)-A$$

$C+D=0$ $\qquad \boxed{D=+1}$
$B-C=0$ $\qquad \boxed{C=-1}$
$A-B=2$ $\qquad \boxed{B=-1}$
$-A=-1$ $\qquad \Rightarrow \boxed{A=1}$

$$\int\frac{2x-1}{x^3(x-1)}dx=\int\frac{1}{x^3}dx+\int\frac{-1}{x^2}dx+\int\frac{-1}{x}dx+\int\frac{1}{x-1}dx$$
$$=-\frac{1}{2x^2}-\left(-\frac{1}{x}\right)-\ln|x|+ln|x-1|=-\frac{1}{2x^2}+\frac{1}{x}-\ln|x|+\ln|x-1|+C$$

33. $\boxed{\int\frac{x-1}{x^2(x+1)^2}dx}$ $\qquad \frac{x-1}{x^2(x+1)^2}=\frac{A}{x^2}+\frac{B}{x}+\frac{C}{(x+1)^2}+\frac{D}{x+1} \qquad /x^2(x+1)^2$

$$x-1=A(x+1)^2+Bx(x+1)^2+Cx^2+Dx^2(x+1)$$
$$x-1=A(x^2+2x+1)+Bx(x^2+2x+1)+Cx^2+Dx^3+Dx^2$$
$$x-1=Ax^2+2Ax+A+Bx^3+2Bx^2+Bx+Cx^2+Dx^3+Dx^2$$
$$x-1=x^3(B+D)+x^2(A+2B+C+D)+x(2A+B)+A$$

$B+D=0\Rightarrow\boxed{D=-3}$

$A+2B+C+D=0\Rightarrow\boxed{C=-2}$

$2A+B=1\Rightarrow\boxed{B=3}$

$\boxed{A=-1}$

$$\int\frac{x-1}{x^2(x+1)^2}dx=\int\frac{A}{x^2}dx+\int\frac{B}{x}dx+\int\frac{C}{(x+1)^2}dx+\int\frac{D}{x+1}dx$$
$$=\int\frac{-1}{x^2}dx+\int\frac{3}{x}dx+\int\frac{-2}{(x+1)^2}dx+\int\frac{-3}{x+1}dx$$

$$\int\frac{-2}{(x+1)^2}dx=-2\int\frac{dx}{(x+1)^2}\Rightarrow\begin{matrix}x+1=t\\dx=dt\end{matrix}=-2\int\frac{1}{t^2}dt=-2\left(-\frac{1}{t}\right)=\frac{2}{t}=\frac{2}{x+1}$$

$$=-\left(-\frac{1}{x}\right)+3\ln|x|+\frac{2}{x+1}-3\ln|x+1|=\frac{1}{x}+3\ln|x|+\frac{2}{x+1}-3\ln|x+1|+C$$

34. $\boxed{\int\frac{2x+1}{x^3+1}dx}$ $\qquad \frac{2x+1}{(x+1)\underbrace{(x^2-x+1)}_{x_{1,2}=\frac{1\pm\sqrt{1-4\cdot1}}{2}}}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1} \qquad /(x+1)(x^2-x+1)$

$$2x+1=A(x^2-x+1)+(Bx+C)(x+1)$$
$$2x+1=Ax^2-Ax+A+Bx^2+Bx+Cx+C$$
$$2x+1=x^2(A+B)+x(-A+B+C)+A+C$$
$$A+B=0$$

$-A+B+C=2$

$A+C=1$

$\boxed{A}\ \lim_{x\to-1}\frac{2x+1}{x^2-x+1}=\frac{-2+1}{1+1+1}=\boxed{-\frac{1}{3}}\qquad \boxed{B=\frac{1}{3}}\qquad \frac{1}{3}+\frac{1}{3}+C=2\Rightarrow\boxed{C=\frac{4}{3}}$

$$\int\frac{2x+1}{(x+1)(x^2-x+1)}dx=\int\frac{A}{x+1}dx+\int\frac{Bx+C}{x^2-x+1}dx$$

$$I=\int\frac{-\frac{1}{3}}{x+1}dx+\int\frac{\frac{1}{3}x+\frac{4}{3}}{x^2-x+1}dx=-\frac{1}{3}\int\frac{1}{x+1}dx+\frac{1}{3}\int\frac{x+4}{x^2-x+1}dx=-\frac{1}{3}\ln|x+1|+\frac{1}{3}\cdot I_1$$

$I_1=\int\frac{x+4}{x^2-x+1}dx\qquad \begin{matrix}A=1\\B=4\end{matrix}\qquad \begin{matrix}a=1\\b=-1\\c=1\end{matrix}$

$$I_1=\frac{1}{2\cdot1}\ln|x^2-x+1|+\left(4-\frac{1(-1)}{2\cdot1}\right)\cdot\int\frac{1}{x^2-x+1}dx$$

$$I_1=\frac{1}{2}\ln|x^2-x+1|+\frac{9}{2}\cdot\int\frac{1}{x^2-x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}dx$$

$$I_1=\frac{1}{2}\ln|x^2-x+1|+\frac{9}{2}\int\frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}dx\qquad \left|\begin{matrix}x-\frac{1}{2}=t\\dx=dt\end{matrix}\right|$$

$$I_1=\frac{1}{2}\ln|x^2-x+1|+\frac{9}{2}\int\frac{1}{t^2-\left(\frac{\sqrt{3}}{2}\right)^2}dx=\frac{1}{2}\ln|x^2-x+1|+\frac{9}{2}\frac{1}{\frac{\sqrt{3}}{2}}arctg\frac{t}{\frac{\sqrt{3}}{2}}$$

$$I_1=\frac{1}{2}\ln|x^2-x+1|+\frac{9}{\sqrt{3}}arctg\frac{2\left(x-\frac{1}{2}\right)}{\sqrt{3}}=\frac{1}{2}\ln|x^2-x+1|+\frac{9}{\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}$$

$$I=-\frac{1}{3}\ln|x+1|+\frac{1}{3}\left(\frac{1}{2}\ln|x^2-x+1|+\frac{9}{\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}\right)$$

$$I=-\frac{1}{3}\ln|x+1|+\frac{1}{6}\ln|x^2-x+1|+\frac{1}{3}\cdot\frac{9}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}$$

$$\underline{\underline{I=-\frac{1}{3}\ln|x+1|+\frac{1}{6}\ln|x^2-x+1|+\sqrt{3}arctg\frac{2x-1}{\sqrt{3}}+C}}$$

35. $\boxed{\int\frac{1}{x^4-1}dx}=\int\frac{1}{(x-1)(x+1)(x^2+1)}dx=\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1}$

$1=A(x+1)(x^2+1)+B(x-1)(x^2+1)+(Cx+D)(x^2-1)$

$1=A(x^3+x+x^2+1)+B(x^3+x-x^2-1)+Cx^3-Cx+Dx^2-D$

$1=Ax^3+Ax+Ax^2+A+Bx^3+Bx-Bx^2-B+Cx^3-Cx+Dx^2-D$

$1=x^3(A+B+C)+x^2(A-B+D)+x(A+B-C)+A-B-D$

$A+B+C=0$

$A-B+D=0\qquad \Rightarrow\frac{1}{4}-\frac{1}{4}+C=0\Rightarrow\boxed{C=0}$

$A+B-C=0\qquad \Rightarrow\frac{1}{4}-\left(-\frac{1}{4}\right)+D=0\Rightarrow\boxed{D=-\frac{1}{2}}$

$A-B-D=0$

$\boxed{A}\ \lim_{x\to 1}\frac{1}{(x+1)(x^2+1)}=\frac{1}{2\cdot 2}=\boxed{\frac{1}{4}}$

$\boxed{B}\ \lim_{x\to -1}\frac{1}{(x-1)(x^2+1)}=\frac{1}{-2\cdot 2}=\boxed{-\frac{1}{4}}$

$$\int\frac{1}{(x-1)(x+1)(x^2+1)}dx=\int\frac{A}{x-1}dx+\int\frac{B}{x+1}dx+\int\frac{Cx+D}{x^2+1}dx$$

$$=\int\frac{\frac{1}{4}}{x-1}dx+\int\frac{-\frac{1}{4}}{x+1}dx+\int\frac{0\cdot x-\frac{1}{2}}{x^2+1}dx$$

$$=\frac{1}{4}\ln|x-1|-\frac{1}{4}\ln|x+1|-\frac{1}{2}arctg\ x+C$$

Тип IV

36. $\boxed{\int\frac{1}{3x+\sqrt[3]{x^2}}dx}=\int\frac{1}{3x+\sqrt[3]{x}^2}dx\qquad\boxed{\begin{matrix}x=t^3\\ dx=3t^2dt\end{matrix}}$

$$=\int\frac{1}{3t^3+\sqrt[3]{t^3}^2}\cdot 3t^2dt=3\int\frac{1}{3t^3+t^2}t^2dt=3\int\frac{1}{t^2(3t+1)}t^2dt=3\int\frac{1}{3t+1}dt$$

$\boxed{\begin{matrix}3t+1=k\\ 3dt=dk\\ dt=\frac{dk}{3}\end{matrix}}=3\int\frac{1}{k}\frac{dk}{3}=\ln k=\ln|3t+1|=\ln\left|3\sqrt[3]{x}+1\right|+C$

37. $\boxed{\int\frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx}\qquad\boxed{\begin{matrix}\text{смена}\\ x=t^6\\ dx=6t^5dt\end{matrix}}\qquad\boxed{\sqrt[m]{x^n}=\sqrt[m]{x}^n}$

$$\int\frac{t^6+\sqrt[3]{t^6}^2+\sqrt[6]{t^6}}{t^6(1+\sqrt[3]{t^6})}\cdot 6t^5dt=6\int\frac{t^6+t^4+t}{t(1+t^2)}dt=6\int\frac{t(t^5+t^3+1)}{t(t^2+1)}dt=6\int\frac{t^5+t^3+1}{t^2+1}dt$$

$\boxed{\begin{matrix}t^5+t^3+1:\ t^2+1=t^3+\frac{1}{t^2+1}\\ \underline{t^5+t^3}\\ 1\to\text{остатак}\end{matrix}}\ 6\int\left(t^3+\frac{1}{t^2+1}\right)dt=6\left[\int t^3dt+\int\frac{1}{t^2+1}dt\right]$

$$6\left[\frac{t^4}{4}+arctg\ t\right]=\frac{3}{2}\left(\sqrt[6]{x}\right)^4+6arctg\ \sqrt[6]{x}+C$$

38. $\boxed{\int\frac{\sqrt{2x-3}}{\sqrt[3]{2x-3}+1}dx}\qquad\boxed{\begin{matrix}2x-3=t^6\\ 2dx=6t^5dt\\ dx=3t^5dt\end{matrix}}$

$$\int\frac{\sqrt{t^6}}{\sqrt[3]{t^6}+1}3t^5dt=3\int\frac{t^3}{t^2+1}t^5dt=3\int\frac{t^8}{t^2+1}dt=3\int\left(t^6-t^4+t^2-1+\frac{1}{t^2+1}\right)dt$$

$$3\left[\int t^6dt-\int t^4dt+\int t^2dt-\int dt+\int\frac{1}{t^2+1}dt\right]=3\left[\frac{t^7}{7}-\frac{t^5}{5}+\frac{t^3}{3}-t+arctg\ t\right]$$

$t=\sqrt[6]{2x-3}$

$$3\left[\frac{\left(\sqrt[6]{2x-3}\right)^7}{7}-\frac{\sqrt[6]{2x-3}^{\,5}}{5}+\frac{\sqrt[6]{2x-3}^{\,3}}{3}-\sqrt[6]{2x-3}+arctg\ \sqrt[6]{2x-3}\right]+C$$

$$t^8 : t^2+1 = t^6 - t^4 + t^2 - 1 + \frac{1}{t^2+1}$$

$$\underline{-t^8 \pm t^6}$$

$$-t^6$$

$$\underline{\mp t^6 \mp t^4}$$

$$+t^4$$

$$\underline{\pm t^4 \pm t^2}$$

$$-t^2$$

$$\underline{\mp t^2 \mp 1}$$

$$1$$

39. $\boxed{\int \frac{\sqrt{x-1}}{x}dx}$ $\boxed{\begin{array}{l} x-1=t^2 \\ x=t^2+1 \\ dx=2tdt \end{array}}$ $\int \frac{\sqrt{t^2}}{t^2+1}\cdot 2tdt = 2\int \frac{t^2}{t^2+1}dt$

$$\boxed{\begin{array}{l} t^2 : t^2+1 = 1+\frac{-1}{t^2+1} \\ \underline{-t^2 \pm 1} \\ \quad -1 \end{array}} = 2\int\left(1+\frac{-1}{t^2+1}\right)dt = 2\int dt - 2\int\frac{1}{t^2+1}dt = 2t - 2arctg\, t$$

$t=\sqrt{x-1}$ $\quad = \underline{\underline{2\sqrt{x-1} + 2arctg\,\sqrt{x-1} + C}}$

40. $\boxed{\int \frac{1}{(x-1)^2}\sqrt{\frac{x+1}{x-1}}dx}$

$$dx = \frac{2t(t^2-1)-(t^2+1)\cdot 2t}{(t^2-1)^2}dt \qquad \boxed{\frac{x+1}{x-1}=t^2}$$

$$dx = \frac{2t(t^2-1-t^2-1)}{(t^2-1)^2}dt \qquad \boxed{dx = \frac{-4t}{(t^2-1)^2}dt}$$

$x+1 = t^2(x-1)$

$x+1 = xt^2 - t^2$

$x - xt^2 = -t^2 - 1 \quad /-1$

$xt^2 - x = t^2+1$

$x(t^2-1) = t^2+1 \Longrightarrow \boxed{x = \frac{t^2+1}{t^2-1}}$

$$\int \frac{1}{\left(\frac{t^2+1}{t^2-1}\right)^2}\sqrt{t^2}\cdot\frac{-4t}{(t^2-1)^2}dt = \int \frac{1}{\left(\frac{t^2+1-t^2+1}{t^2-1}\right)^2}\frac{-4t^2}{(t^2-1)^2}dt$$

$$= -4\int \frac{1}{\frac{2^2}{(t^2-1)^2}}\cdot\frac{t^2}{(t^2-1)^2}dt = -\frac{4}{4}\int t^2dt = -\frac{t^3}{3} = \underline{\underline{-\frac{\sqrt{\frac{x+1}{x-1}}^3}{3} + C}}$$

41. $\boxed{\int \frac{4}{(x-1)^2}\sqrt[3]{\frac{1-x}{1+x}}dx}$

$$dx = \frac{-3t^2(1+t^3)-(1-t^3)3t^2}{(1+t^3)^2}dt \qquad \boxed{\frac{1-x}{1+x}=t^3}$$

$$dx = \frac{3t^2(-1-t^3-1+t^3)}{(1+t^3)^2}dt \qquad \boxed{dx = \frac{-6t^2}{(1+t^3)^2}dt}$$

$1-x = t^3(1+x)$

$1-x = t^3 + xt^3$

$-x - xt^3 = t^3 - 1 \quad /-1$

$x + xt^3 = 1 - t^3$

$x(1+t^3) = 1-t^3$

$\boxed{x = \frac{1-t^3}{1+t^3}}$

$$\int \frac{4}{\left(\frac{1-t^3}{1+t^3}-1\right)^2}\cdot\sqrt[3]{t^3}\cdot\frac{-6t^2}{(1+t^3)^2}dt = 4\int \frac{1}{\left(\frac{1-t^3-1-t^3}{1+t^3}\right)^2}\cdot\frac{-6t^3}{(1+t^3)^2}dt$$

$$-24\int \frac{1}{\frac{(-2t^3)^2}{(1+t^3)^2}}\cdot\frac{t^3}{(1+t^3)^2}dt = -\frac{24}{4}\int\frac{1}{t^6}t^3dt = -6\int\frac{1}{t^3}dt = -6\cdot\frac{1}{-2t^2} = 3\cdot\frac{1}{t^2}$$

$$3\cdot\frac{1}{\left(\sqrt[3]{\frac{1-x}{1+x}}\right)^2} = 3\left(\sqrt[3]{\frac{1+x}{1-x}}\right)^2 + C$$

42. $\boxed{\int\frac{1}{x\cdot\sqrt{x^2-9}}dx}$ $\quad\left| \begin{array}{c} x=\frac{1}{t} \\ t=\frac{1}{x} \\ dx=-\frac{1}{t^2}dt \end{array} \right|$ $\quad\int\frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2}-9}}\left(-\frac{1}{t^2}\right)dt = -\int\frac{1}{\sqrt{\frac{1-9t^2}{t^2}}}\frac{1}{t}dt$

$$-\int\frac{1}{\frac{\sqrt{1-9t^2}}{\sqrt{t^2}}}\frac{1}{t}dt = -\int\frac{1}{\sqrt{1-9t^2}}dt = -\int\frac{1}{\sqrt{1-(3t)^2}}dt \qquad \left| \begin{array}{c} 3t=k \\ 3dt=dk \\ dt=\frac{dk}{3} \end{array} \right|$$

$$-\int\frac{1}{\sqrt{1-k^2}}\frac{dk}{3} = -\frac{1}{3}\int\frac{1}{\sqrt{1-k^2}}dk = -\frac{1}{3}\arcsin k = -\frac{1}{3}\arcsin 3t = -\frac{1}{3}\arcsin\frac{3}{x} + C$$

43. $\boxed{\int\frac{1}{x^2\cdot\sqrt{x^2+4}}dx}$ $\quad\left| \begin{array}{c} x=\frac{1}{t} \\ t=\frac{1}{x} \\ dx=-\frac{1}{t^2}dt \end{array} \right|$ $\quad\int\frac{1}{\frac{1}{t^2}\sqrt{\frac{1}{t^2}+4}}\left(-\frac{1}{t^2}\right)dt = -\int\frac{1}{\sqrt{\frac{1+4t^2}{t^2}}}dt$

$$-\int\frac{1}{\frac{\sqrt{1+4t^2}}{\sqrt{t^2}}}dt = -\int\frac{t}{\sqrt{1+4t^2}}dt \qquad \left| \begin{array}{ll} 1+4t^2=k^2 & dt=\frac{2kdk}{8t} \\ k=\sqrt{1+4t^2} & \\ 8tdt=2kdk & dt=\frac{kdk}{4t} \end{array} \right|$$

$$-\int\frac{t}{\sqrt{k^2}}\frac{kdk}{4t} = -\frac{1}{4}\int dk = -\frac{1}{4}k = -\frac{1}{4}\sqrt{1+4t^2} = -\frac{1}{4}\sqrt{1+\frac{4}{x^2}} + C$$

Парцијална интеграција

44. $\boxed{\int(x^2-1)e^xdx}$ $\quad\left| \begin{array}{ll} & dv=e^xdx \\ u=x^2-1 & v=\int e^xdx \\ du=2xdx & \\ & v=e^x \end{array} \right|$

$$= (x^2-1)e^x - \int e^x 2xdx = (x^2-1)e^x - 2\int xe^xdx = \left| \begin{array}{ll} & dv=e^xdx \\ u=x & v=\int e^xdx \\ du=dx & \\ & v=e^x \end{array} \right| \begin{array}{c} xe^x-\int e^xdx \\ xe^x-e^x=e^x(x-1) \end{array}$$

$$= (x^2-1)e^x - 2e^x(x-1) = e^x(x^2-1-2x+2) = e^x(x^2-2x+1) = e^x(x-1)^2 + C$$

45. $\boxed{\int \frac{x}{\sin^2 x}dx}$

$$\left| \begin{matrix} u = x & dv = \frac{1}{\sin^2 x}dx \\ du = dx & v = \int \frac{1}{\sin^2 x}dx \\ & v = -ctg\, x \end{matrix} \right| = x \cdot (-ctg\, x) - \int -ctg\, x\, dx$$

$$= -x\, ctg\, x + \int \frac{\cos x}{\sin x}dx = \left| \begin{matrix} \sin x = t \\ \cos x\, dx = dt \\ dx = \frac{dt}{\cos x} \end{matrix} \right| = -x\, ctg\, x + \int \frac{\cos x}{t} \cdot \frac{dt}{\cos x} = -x\, ctg\, x + \int \frac{1}{t}dt$$

$$-x\, ctg\, x + \ln|t| = \underline{\underline{-x\, ctg\, x + \ln|\sin x| + C}}$$

46. $\boxed{\int x^2 \cdot \sin x\, dx}$

$$\left| \begin{matrix} u = x^2 & dv = \sin x\, dx \\ du = 2x\, dx & v = \int \sin x\, dx \\ & v = -\cos x \end{matrix} \right| = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x\, dx$$

$$= x^2 \cos x + 2\int x \cos x\, dx = \left| \begin{matrix} u = x & dv = \cos x\, dx \\ du = dx & v = \int \cos x\, dx \\ & v = \sin x \end{matrix} \right| \begin{matrix} x \cdot \sin x - \int \sin x\, dx \\ x \cdot \sin x - (-\cos x) \end{matrix}$$

$$= x^2 \cos x + 2(x \sin x + \cos x) = \underline{\underline{x^2 \cos x + 2x \sin x + 2\cos x + C}}$$

47. $\boxed{\int x \cdot e^{2x}dx}$

$$\left| \begin{matrix} u = x & dv = e^{2x}dx & \left| \begin{matrix} 2x = t \\ 2dx = dt \\ dx = \frac{dt}{2} \end{matrix} \right| & \begin{matrix} v = \int e^t \frac{dt}{2} \\ v = \frac{1}{2}e^t \\ v = \frac{1}{2}e^{2x} \end{matrix} \\ du = dx & v = \int e^{2x}dx & & \end{matrix} \right|$$

$$= x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} \cdot dx = \frac{1}{2}xe^{2x} - \frac{1}{2} \cdot \frac{1}{2}e^{2x} = \underline{\underline{\frac{1}{2}e^{2x}\left(x - \frac{1}{2}\right) + C}}$$

48. $\boxed{\int (x^2 - 3x + 2)e^x dx}$

$$\left| \begin{matrix} u = x^2 - 3x + 2 & dv = e^x dx \\ du = (2x - 3)dx & v = \int e^x dx \\ & v = e^x \end{matrix} \right|$$

$$= (x^2 - 3x + 2)e^x - \int e^x(2x - 3)dx = \left| \begin{matrix} u = 2x - 3 & dv = e^x dx \\ du = 2dx & v = \int e^x dx \\ & v = e^x \end{matrix} \right| \begin{matrix} (2x - 3)e^x - \int e^x 2dx \\ (2x - 3)e^x - 2e^x \\ e^x(2x - 3 - 2) = e^x(2x - 5) \end{matrix}$$

$$= (x^2 - 3x + 2)e^x - e^x(2x - 5) = e^x(x^2 - 3x + 2 - 2x + 5) = \underline{\underline{e^x(x^2 - 5x + 7) + C}}$$

49. $\boxed{\int x \cdot \ln x\, dx}$

$$\left| \begin{matrix} u = \ln x & dv = x dx \\ du = \frac{1}{x}dx & v = \int x\, dx \\ & v = \frac{x^2}{2} \end{matrix} \right|$$

$$= \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\, dx = \frac{x^2}{2}\ln x - \frac{1}{2} \cdot \frac{x^2}{2} = \underline{\underline{\frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + C}}$$

50. $\boxed{\int x^2 \cdot arctg\, x\, dx}$

$$\left| \begin{matrix} u = arctg\, x & dv = x^2 dx \\ du = \frac{1}{x^2 + 1}dx & v = \int x^2\, dx \\ & v = \frac{x^3}{3} \end{matrix} \right|$$

$$= arctg\, x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x^2+1} dx = \frac{x^3}{3} arctg\, x - \frac{1}{3}\int \frac{x^3}{x^2+1} dx$$

$$x^3 : x^2 + 1 = x - \frac{x}{x^2+1}$$

$$\underline{-x^3 \pm x}$$

$$-x$$

$$\int x\, dx - \int \frac{x}{x^2+1} dx = \left| \begin{matrix} x^2+1 = t \\ 2xdx = dt \\ dx = \frac{dt}{2x} \end{matrix} \right| = \begin{matrix} \frac{x^2}{2} - \int \frac{x}{t} \cdot \frac{dt}{2x} \\ \frac{x^2}{2} - \frac{1}{2}\ln t \\ \frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| \end{matrix}$$

$$= \underline{\underline{\frac{x^3}{3} arctg\, x - \frac{1}{3}\left(\frac{x^2}{2} - \frac{1}{2}\ln|x^2+1|\right) + C}}$$

51. $\boxed{\int \ln(x^2+1)\ dx}$ $\left| \begin{matrix} u = \ln|x^2+1| \\ du = \frac{1}{x^2+1} 3xdx \end{matrix} \quad \begin{matrix} dv = dx \\ v = \int dx \\ v = x \end{matrix} \right|$

$$= \ln|x^2+1| \cdot x - \int x \cdot \frac{2x}{x^2+1} dx = x \cdot \ln|x^2+1| - 2\int \frac{x^2}{x^2+1} dx$$

$$x^2 : x^2 + 1 = 1 - \frac{1}{x^2+1}$$

$$\underline{-x^2 \pm 1}$$

$$-1$$

$$\int \left(1 - \frac{1}{x^2+1}\right) dx$$

$$\int dx - \int \frac{1}{x^2+1} dx = x - arctg\, x$$

$$= \underline{\underline{x \cdot \ln|x^2+1| - 2 \cdot (x - arctg\, x) + C}}$$

52. $\boxed{\int e^{2x} \cdot \cos 3x\ dx}$ $\left| \begin{matrix} u = \cos 3x \\ du = -3\sin 3x\, dx \end{matrix} \quad \begin{matrix} dv = e^{2x}dx \\ v = \int e^{2x}\, dx \end{matrix} \left| \begin{matrix} 2x = t \\ 2dx = dt \\ dx = \frac{dt}{2} \end{matrix} \right| \begin{matrix} v = \int e^t \frac{dt}{2} \\ v = \frac{1}{2}e^t \\ v = \frac{1}{2}e^{2x} \end{matrix} \right|$

$$I = \cos 3x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}(-3\sin 3x)dx$$

$$I = \frac{1}{2}e^{2x} \cdot \cos 3x + \frac{3}{2}\underbrace{\int e^{2x}\sin 3x\, dx}_{I_1} \qquad \begin{matrix} u = \sin 3x \\ du = 3\cos 3x\, dx \end{matrix} \quad \begin{matrix} dv = e^{2x}dx \\ \vdots \\ v = \frac{1}{2}e^{2x} \end{matrix}$$

$$I_1 = \sin 3x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} \cdot 3\cos 3x\, dx$$

$$I_1 = \frac{1}{2}e^{2x} \cdot \sin 3x - \frac{3}{2}\underbrace{\int e^{2x}\cos 3x\, dx}_{I}$$

$$I = \frac{1}{2}e^{2x} \cdot \cos 3x + \frac{3}{2} \cdot I_1$$

$$I = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2} \cdot \left(\frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}I\right)$$

$$I = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}I$$

$$\frac{9}{4}I + I = \frac{1}{2}e^{2x}\left(\cos 3x + \frac{3}{2}\sin 3x\right)$$

$$I = \frac{4}{13} \cdot \frac{1}{2}e^{2x}\left(\cos 3x + \frac{3}{2}\sin 3x\right)$$

$$\underline{\underline{I = \frac{2}{13}e^{2x}\left(\cos 3x + \frac{3}{2}\sin 3x\right) + C}}$$

53. $\boxed{\int e^{3x}\cdot \sin 2x\ dx}$

$$\left|\begin{matrix} u=\sin 2x & dv=e^{3x}dx \\ du=2\cos 2x\,dx & v=\int e^{3x}\,dx\end{matrix}\ \left|\begin{matrix}3x=t\\ 3dx=dt\\ dx=\dfrac{dt}{3}\end{matrix}\right|\ \begin{matrix} v=\int e^t\dfrac{dt}{3}\\ v=\dfrac{1}{3}e^t \\ v=\dfrac{1}{3}e^{2x}\end{matrix}\right|$$

$$I=\sin 2x\cdot\frac{1}{3}e^{3x}-\int\frac{1}{3}e^{3x}\cdot 2\cos 2x\,dx$$

$$I=\sin 2x\cdot\frac{1}{3}e^{3x}-\frac{2}{3}\underbrace{\int e^{3x}\cdot\cos 2x\,dx}_{I_1}\qquad \left|\begin{matrix}u=\cos 2x & dv=e^{3x}dx\\ & \vdots \\ du=-2\sin 2x\,dx & v=\dfrac{1}{3}e^{3x}\end{matrix}\right|$$

$$I_1=\cos 2x\cdot\frac{1}{3}e^{3x}-\int\frac{1}{3}e^{3x}(-2\sin 2x)dx$$

$$I_1=\frac{1}{3}\cos 2x\cdot e^{3x}-\frac{2}{3}\underbrace{\int e^{3x}\sin 2x\,dx}_{I}$$

$$I=\frac{1}{3}e^{3x}\sin 2x-\frac{2}{3}\left(\frac{1}{3}e^{3x}\cos 2x-\frac{2}{3}I\right)$$

$$I=\frac{1}{3}e^{3x}\sin 2x-\frac{2}{9}e^{3x}\cos 2x-\frac{4}{9}I$$

$$\frac{4}{9}I+I=\frac{1}{3}e^{3x}\left(\sin 2x-\frac{2}{3}\cos 2x\right)$$

$$I=\frac{9}{13}\cdot\frac{1}{3}e^{3x}\left(\sin 2x-\frac{2}{3}\cos 2x\right)$$

$$\underline{\underline{I=\frac{3}{13}e^{3x}\left(\sin 2x-\frac{2}{3}\cos 2x\right)+C}}$$

54. $\boxed{\int \sin(\ln x)\ dx}$

$$\begin{matrix}\ln x=t\Longrightarrow\\ \dfrac{1}{x}dx=dt\\ dx=xdt\end{matrix}\qquad \begin{matrix}\ln x=t\\ e^{\ln x}=e^t\\ \boxed{x=e^t}\end{matrix}\qquad \begin{matrix}\int\sin t\cdot xdt\\ I=\int\sin t\cdot e^tdt\end{matrix}\qquad \left|\begin{matrix}u=\sin t & dv=e^tdt\\ du=\cos t\,dt & v=\int e^tdt\\ & v=e^t\end{matrix}\right|$$

$$I=\sin t\cdot e^t-\underbrace{\int e^t\cdot\cos t\,dt}_{I_1}\qquad\left|\begin{matrix}u=\cos t & dv=e^tdt\\ du=-\sin t\,dt & v=\int e^tdt\\ & v=e^t\end{matrix}\right|$$

$$I_1=\cos t\cdot e^t-\int e^t(-\sin t)dt$$

$$I_1=e^t\cos t+\underbrace{\int e^t\sin t\,dt}_{I}$$

$$I=e^t\sin t-(e^t\cos t+I)$$

$$I=e^t\sin t-e^t\cos t-I$$

$$2I=e^t(\sin t-\cos t)$$

$$I=\frac{1}{2}e^{\ln x}(\sin\ln x-\cos\ln x)$$

$$\underline{\underline{I=\frac{1}{2}x(\sin(\ln x)-\cos(\ln x))+C}}$$

55. $\boxed{\int\sqrt{x^2+4}\,dx}\cdot\dfrac{\sqrt{x^2+4}}{\sqrt{x^2+4}}$

$$I=\int\frac{x^2+4}{\sqrt{x^2+4}}dx=\int\frac{x^2}{\sqrt{x^2+4}}dx+\int\frac{4}{\sqrt{x^2+4}}dx$$

$$I=\int x\cdot\frac{x}{\sqrt{x^2+4}}dx+4\int\frac{1}{\sqrt{x^2+4}}dx$$

$$\left|\begin{matrix}u=x & dv=\dfrac{x}{\sqrt{x^2+4}}dx\\ du=dx & v=\int\dfrac{x}{\sqrt{x^2+4}}dx\end{matrix}\ \left|\begin{matrix}x^2+4=t^2\\ t=\sqrt{x^2+4}\\ 2xdx=2tdt\\ dx=\dfrac{tdt}{x}\end{matrix}\right|\ \begin{matrix}v=\int\dfrac{x}{\sqrt{t^2}}\cdot\dfrac{tdt}{x}\\ v=\int dt\\ v=t\\ v=\sqrt{x^2+4}\end{matrix}\right|$$

$$I = x\cdot\sqrt{x^2+4} - \underbrace{\int \sqrt{x^2+4}dx}_{I} + 4\cdot\ln\left|x+\sqrt{x^2+4}\right|$$

$$I = x\cdot\sqrt{x^2+4} - I + 4\ln\left|x+\sqrt{x^2+4}\right|$$

$$2I = x\cdot\sqrt{x^2+4} + 4\ln\left|x+\sqrt{x^2+4}\right|$$

$$\underline{\underline{I = \frac{x}{2}\sqrt{x^2+4} + 2\ln\left|x+\sqrt{x^2+4}\right| + C}}$$

56. $\boxed{\int \frac{x^2}{\sqrt{9-x^2}}\,dx}$ $\quad I = \int \frac{x^2}{\sqrt{9-x^2}}\,dx = \int x\cdot\frac{x}{\sqrt{9-x^2}}\,dx$

$$\left|\begin{matrix} u = x \\ du = dx \end{matrix}\quad \begin{matrix} dv = \frac{x}{\sqrt{9-x^2}}dx \\ v = \int \frac{x}{\sqrt{9-x^2}}dx \end{matrix}\quad \left|\begin{matrix} 9-x^2 = t^2 \\ t = \sqrt{9-x^2} \\ -2xdx = 2tdt \\ dx = \frac{tdt}{-x} \end{matrix}\right|\quad \begin{matrix} v = \int \frac{x}{\sqrt{t^2}}\cdot\frac{tdt}{-x} \\ v = -\int dt \\ v = -t \\ v = -\sqrt{9-x^2} \end{matrix}\right|$$

$$I = x\cdot\left(-\sqrt{9-x^2}\right) - \int -\sqrt{9-x^2}dx$$

$$I = -x\sqrt{9-x^2} + \int \sqrt{9-x^2}dx\cdot\frac{\sqrt{9-x^2}}{\sqrt{9-x^2}}$$

$$I = -x\sqrt{9-x^2} + \int \frac{9-x^2}{\sqrt{9-x^2}}dx$$

$$I = -x\sqrt{9-x^2} + 9\int \frac{1}{\sqrt{9-x^2}}dx - \underbrace{\int \frac{x^2}{\sqrt{9-x^2}}dx}_{I}$$

$$I = -x\sqrt{9-x^2} + 9\cdot\arcsin\frac{x}{3} - I$$

$$2I = -x\sqrt{9-x^2} + 9\arcsin\frac{x}{3}$$

$$\underline{\underline{I = -\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3} + C}}$$

57. $\boxed{\int \sqrt{1-x^2}\,dx}\cdot\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$ $\quad I = \int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{x^2}{\sqrt{1-x^2}}\,dx$

$$I = \arcsin x - \int x\cdot\frac{x}{\sqrt{1-x^2}}\,dx$$

$$\left|\begin{matrix} u = x \\ du = dx \end{matrix}\quad \begin{matrix} dv = \frac{x}{\sqrt{1-x^2}}dx \\ v = \int \frac{x}{\sqrt{1-x^2}}dx \end{matrix}\quad \left|\begin{matrix} 1-x^2 = t^2 \\ -2xdx = 2tdt \\ dx = \frac{tdt}{-x} \end{matrix}\right|\quad \begin{matrix} v = \int \frac{x}{\sqrt{t^2}}\cdot\frac{tdt}{-x} \\ v = -t \\ v = -\sqrt{1-x^2} \end{matrix}\right|$$

$$I = \arcsin x - \left(x\cdot\left(-\sqrt{1-x^2}\right) - \int -\sqrt{1-x^2}dx\right)$$

$$I = \arcsin x + x\sqrt{1-x^2} - \underbrace{\int \sqrt{1-x^2}dx}_{I}$$

$$I = \arcsin x + x\sqrt{1-x^2} - I$$

$$2I = \arcsin x + x\sqrt{1-x^2}$$

$$\underline{\underline{I = \frac{1}{2}\arcsin x + \frac{x}{2}\sqrt{1-x^2} + C}}$$

ТИП IV

58. $\boxed{\int \frac{1}{8-4\sin x+7\cos x}\,dx}$

$$\left|\begin{matrix} t = tg\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \\ dx = \frac{2dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} \end{matrix}\right|$$

$$\int \frac{1}{8-4\cdot\frac{2t}{t^2+1}+7\cdot\frac{1-t^2}{t^2+1}}\cdot\frac{2dt}{t^2+1} = 2\int \frac{1}{\frac{8t^2+8-8t+7-7t^2}{t^2+1}}\,\frac{2dt}{t^2+1} = 2\int \frac{1}{t^2-8t+15}\,dt$$

$$2\int \frac{1}{t^2-8t+4^2-4^2+15}\,dt = 2\int \frac{1}{(t-4)^2-1}\,dt = \left|\begin{matrix} t-4=k \\ dt=dk \end{matrix}\right| = 2\int \frac{1}{k^2-1}\,dk$$

$$2\cdot\frac{1}{2\cdot 1}\ln\frac{k-1}{k+1} = \ln\frac{t-4-1}{t-4+1} = \ln\left|\frac{tg\frac{x}{2}-5}{tg\frac{x}{2}-3}\right| + C$$

59. $\boxed{\int \frac{2-\sin x}{2+\cos x}\,dx}$ $\left|\begin{matrix} t = tg\frac{x}{2} & \sin x = \frac{2t}{t^2+1} \\ dx = \frac{2dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} \end{matrix}\right|$

$$\int \frac{2-\frac{2t}{t^2+1}}{2+\frac{1-t^2}{t^2+1}}\cdot\frac{2dt}{t^2+1} = 2\cdot\int \frac{\frac{2t^2+2-2t}{t^2+1}}{\frac{2t^2+2+1-t^2}{t^2+1}}\,\frac{dt}{t^2+1} = 2\cdot\int \frac{2t^2-2t+2}{t^2+3}\cdot\frac{dt}{t^2+1}$$

$$\frac{2t^2-2t+2}{(t^2+3)(t^2+1)} = \frac{At+B}{t^2+3}+\frac{Ct+D}{t^2+1} \qquad /(t^2+3)(t^2+1)$$

$$2t^2-2t+2 = (At+B)(t^2+1)+(Ct+D)(t^2+3)$$

$$2t^2-2t+2 = At^3+At+Bt^2+B+Ct^3+3Ct+Dt^2+3D$$

$$2t^2-2t+2 = t^3(A+C)+t^2(B+D)+t(A+3C)+B+3D$$

$A+C=0$ $\quad$ $A+C=0\ \ /(-1)$

$B+D=2$ $\quad$ $\underline{A+3C=-2}$

$A+3C=-2$ $\quad$ $-A-C=0$

$B+3D=2$ $\quad$ $\underline{A+3C=-2}$ $\Big\} = 2C-2 \Rightarrow \boxed{C=-1}$ $\boxed{A=1}$

$B+D=2\ \ /(-1)$

$\underline{B+3D=2}$

$-B-D=-2$

$\underline{B+3D=2}$ $\Big\} 2D=0 \Rightarrow \boxed{D=0}$ $\boxed{B=2}$

$$2\cdot\int \frac{2t^2-2t+2}{(t^2+3)(t^2+1)}\,dt = 2\left(\int \frac{At+B}{t^2+3}\,dt+\int\frac{Ct+D}{t^2+1}\,dt\right) = 2\left(\int\frac{t+2}{t^2+3}\,dt+\int\frac{-t}{t^2+1}\,dt\right)$$

$\begin{matrix} A=1 \\ B=2 \end{matrix}$ $\begin{matrix} a=1 \\ b=0 \\ c=3 \end{matrix}$ $\left|\begin{matrix} t^2+1=k \\ 2tdt=dk \\ dt=\frac{dk}{2t} \end{matrix}\right|$ $2\cdot\left(\frac{1}{2}\ln|t^2+3|+\left(1-\frac{1\cdot 0}{2\cdot 1}\right)\cdot\int\frac{1}{t^2+3}\,dt-\int\frac{t}{k}\cdot\frac{dk}{2t}\right)$

$$2\left(\frac{1}{2}\ln|t^2+3|+\int\frac{1}{t^2+\sqrt{3}^2}\,dt-\ln k\right) = 2\left(\frac{1}{2}\ln|t^2+3|+\frac{1}{\sqrt{3}}arctg\frac{t}{\sqrt{3}}-\ln|t^2+1|\right)$$

$$\ln\left|tg\frac{x}{2}+3\right|+\frac{2}{\sqrt{3}}arctg\frac{tg\frac{x}{2}}{\sqrt{3}}-2\ln\left|tg\frac{x}{2}+1\right|+C$$

60. $\boxed{\int \frac{1}{4-3\cos^2 x+5\sin^2 x}\,dx}$ $\left|\begin{matrix} t = tgx & \sin^2 x = \frac{t^2}{t^2+1} \\ dx = \frac{dt}{t^2+1} & \cos^2 x = \frac{1}{t^2+1} \end{matrix}\right|$

$$\int \frac{1}{4-3\cdot\frac{t^2}{t^2+1}+5\cdot\frac{1}{t^2+1}}\cdot\frac{dt}{t^2+1} = \int\frac{1}{\frac{4t^2+4-3+5t^2}{t^2+1}}\cdot\frac{dt}{t^2+1} = \int\frac{1}{9t^2+1}\,dt$$

$$\int \frac{1}{(3t)^2+1}\,dt = \left| \begin{matrix} 3t = k \\ 3dt = dk \\ dt = \frac{dk}{3} \end{matrix} \right| = \int \frac{1}{k^2+1}\frac{dk}{3} = \frac{1}{3} arctg\, k = \frac{1}{3} arctg\, 3t = \underline{\underline{\frac{1}{3} arctg\, 3 \cdot tg\, x + C}}$$

61. $\boxed{\int \frac{1}{\sin^2 x + 3\sin x \cdot \cos x - \cos^2 x}\,dx}$ $\left| \begin{matrix} t = tgx & \sin^2 x = \frac{t^2}{t^2+1} \\ dx = \frac{dt}{t^2+1} & \cos^2 x = \frac{1}{t^2+1} \end{matrix} \right|$

$$\int \frac{1}{\frac{t^2}{t^2+1} + 3\cdot\frac{t}{\sqrt{t^2+1}}\cdot\frac{1}{\sqrt{t^2+1}} - \frac{1}{t^2+1}} \cdot \frac{1}{t^2+1}\,dt = \int \frac{1}{\frac{t^2+3t-1}{t^2+1}} \cdot \frac{1}{t^2+1}\,dt$$

$$\int \frac{1}{t^2+3t-1}\,dt = \int \frac{1}{t^2+3t+\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1}\,dt = \int \frac{1}{\left(t+\frac{3}{2}\right)^2 - \frac{13}{4}}\,dt \quad \left| \begin{matrix} t + \frac{3}{2} = k \\ dt = dk \end{matrix} \right|$$

$$\int \frac{1}{k^2 - \left(\frac{\sqrt{13}}{2}\right)^2}\,dt = \frac{1}{2\cdot\frac{\sqrt{13}}{2}} \ln\frac{k - \frac{\sqrt{13}}{2}}{k + \frac{\sqrt{13}}{2}} = \frac{1}{\sqrt{13}} \ln\frac{t+\frac{3}{2} - \frac{\sqrt{13}}{2}}{t+\frac{3}{2}+\frac{\sqrt{13}}{2}} = \underline{\underline{\frac{1}{\sqrt{13}} \ln\frac{tg\,x + \frac{3-\sqrt{13}}{2}}{tg\,x + \frac{3+\sqrt{13}}{2}}}} + C$$

62. $\boxed{\int \frac{\sin^3 x}{1+\cos^2 x}\,dx}$ $\left| \begin{matrix} \cos x = t \\ -\sin x\,dx = dt \\ dx = \frac{dt}{-\sin x} \end{matrix} \right|$

$$\int \frac{\sin^3 x}{1+t^2} \cdot \frac{dt}{-\sin x} = -\int \frac{\sin^2 x}{1+t^2}dt = \begin{pmatrix} \sin^2 x + \cos^2 x = 1 \\ \sin^2 x = 1 - \cos^2 x \\ \sin^2 x = 1 - t^2 \end{pmatrix} = -\int \frac{1-t^2}{1+t^2}dt = \int \frac{t^2-1}{t^2+1}dt$$

$$t^2 - 1 : t^2 + 1 = 1 - \frac{2}{t^2+1}$$

$$\underline{-t^2 \pm 1}$$

$$-2$$

$$= \int 1\,dt - \int \frac{2}{t^2+1}dt = t - 2arctg\,t$$

$$= \underline{\underline{\cos x - 2arctg(\cos x) + C}}$$

63. $\boxed{\int \frac{\cos^3 x}{4\sin^2 x - 1}\,dx}$ $\left| \begin{matrix} \sin x = t \\ \cos x\,dx = dt \\ dx = \frac{dt}{\cos x} \end{matrix} \right|$ $\int \frac{\cos^3 x}{4\cdot t^2 - 1}\frac{dt}{\cos x}$ $\quad \begin{matrix} \sin^2 x + \cos^2 x = 1 \\ \sin^2 x = 1 - \cos^2 x \end{matrix}$

$$t^2 - 1 : 4t^2 - 1 = \frac{1}{4} - \frac{\frac{3}{4}}{4t^2-1}$$

$$\int \frac{1-t^2}{4t^2-1}\,dt = -\int \frac{t^2-1}{4t^2-1}\,dt$$

$$\underline{-t^2 \mp \frac{1}{4}}$$

$$-\frac{3}{4}$$

$$-\left(\int\left(\frac{1}{4} - \frac{\frac{3}{4}}{4t^2-1}\right)dt\right) = -\int \frac{1}{4}dt + \int \frac{\frac{3}{4}}{4t^2-1}dt = -\frac{1}{4}t + \frac{3}{4}\int \frac{1}{(2t)^2-1}dt$$

$$\left| \begin{matrix} 2t = k \\ 2dt = dk \\ dt = \frac{dk}{2} \end{matrix} \right| = -\frac{1}{4}t + \frac{3}{4}\int \frac{1}{k^2-1}\frac{dk}{2} = -\frac{1}{4}t + \frac{3}{2\cdot4\cdot2}\ln\frac{k-1}{k+1} = -\frac{1}{4}t + \frac{3}{16}\ln\frac{2t-1}{2t+1}$$

$$-\frac{1}{4}\sin x+\frac{3}{16}\ln\left|\frac{2\sin x-1}{2\sin x+1}\right|+C$$