1) $\boxed{\int \frac{8x^2-12x+2}{x^3-2x^2+x-2}dx}$

$x^3-2x^2+x-2 = x^3+x-2x^2-2 = x(x^2+1)-2(x^2+1) = (x^2+1)(x-2)$

Никад није згодно за раставити

$(x^3-2x^2+x-2):(x-2) = x^2+1$

$\underline{\pm x^3 \mp 2x^2}$

$x-2$

$\underline{\pm x \mp 2}$

0	$0-0+0-2\neq 0$
1	$1-2+1-2\neq 0$
-1	$-1-2-2-2\neq 0$
$\boxed{2}$	$8-8+2-2\neq 0$

$\boxed{x^3-2x^2+x-2=(x-2)(x^2+1)}$

$$\frac{8x^2-12x+2}{x^3-2x^2+x-2}=\frac{8x^2-12x+2}{(x-2)(x^2+1)}$$

$$\frac{8x^2-12x+2}{(x-2)(x^2+1)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1} \quad /(x-2)(x^2+1)$$

$$A=\lim_{x\to 2}\frac{8x^2-12x+2}{x^2+1}=\frac{8\cdot 4-12\cdot 2+2}{4+1}=\boxed{\frac{10}{5}}=\boxed{2}$$

$8x^2-12x+2=A(x^2+1)+(x-2)(Bx+C)$

$8x^2-12x+2=Ax^2+A+Bx^2+Cx-2Bx-2C$

$8x^2-12x+2=x^2(A+B)+x(C-2B)+(A-2C)$

$A+B=8 \quad \to B=6$

$C-2B=-12 \quad \to C=0 \quad A=2$

$A-2C=2$

Провера: $\left.\begin{array}{r} 2+6=8 \\ 0-2\cdot 6=-12 \\ 2-2\cdot 0=2 \end{array}\right\}$ $\begin{array}{l} A=2 \\ B=6 \\ C=0 \end{array}$ тачно решење

$$\int \frac{8x^2-12x+2}{x^3-2x^2+x-2}dx=\int\frac{A}{x-2}dx+\int\frac{Bx+C}{x^2+1}dx=\int\frac{2}{x-2}dx+\int\frac{6x+0}{x^2+1}dx$$

$$x^2+1=t, \qquad 2xdx=dt, \qquad dx=\frac{dt}{2x}$$

$$2\int\frac{1}{x-2}dx+\int\frac{6x}{t}\frac{dt}{2x}=2\ln|x-2|+3\ln t=2\ln|x-2|+3\ln|x^2+1|+C$$

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2) $\boxed{\int \frac{21x^2-94x+72}{x^3-7x^2+12x}dx}$ $\qquad x_{1,2}=\frac{7\pm\sqrt{49-48}}{2} \quad \begin{array}{l} x_1=4 \\ x_2=4 \end{array}$

$$\frac{21x^2-94x+72}{x(x^2-7x+12)}=\frac{21x^2-94x+72}{x(x-4)(x-3)}$$

$$\frac{21x^2-94x+72}{x(x-4)(x-3)}=\frac{A}{x}+\frac{B}{x-4}+\frac{C}{x-3}$$

$$A=\lim_{x\to 0}\frac{21x^2-94x+72}{(x-4)(x-3)}=\frac{72}{12}=6$$

$$B=\lim_{x\to 4}\frac{21x^2-94x+72}{x(x-3)}=\frac{21\cdot 4^2-94\cdot 4+72}{4\cdot 1}=8$$

$$C=\lim_{x\to 3}\frac{21x^2-94x+72}{x(x-4)}=\frac{21\cdot 9-94\cdot 3+72}{3\cdot(-1)}=7$$

$$\int\frac{21x^2-94x+72}{x^3-7x^2+12x}dx=\int\left(\frac{A}{x}+\frac{B}{x-4}+\frac{C}{x-3}\right)dx=\int\frac{6}{x}dx+\int\frac{8}{x-4}dx+\int\frac{7}{x-3}dx=$$

$= 6\ln x + 8\ln|x-4| + 7\ln|x-3| + C$

---

3) $\boxed{\int \frac{x\sqrt[3]{x+2}}{x+\sqrt[3]{x+2}}dx}$ $\quad x+2=t^3 \rightarrow x = t^3-2, \quad dx = 3t^2dt, \quad t^3+t-2=0$

$$\int \frac{(t^3-2)\sqrt[3]{t^3}}{t^3-2+\sqrt[3]{t^3}}\cdot 3t^2dt = 3\int \frac{t^6-2t^3}{t^3+t-2}dt$$

$(t^3+t-2):(t-1) = t^3+t+2$

$\underline{\pm t^3 \mp t^2}$

t^2+t-2

$\underline{-t^2 \mp t}$

$2t-2$

$\underline{-2t \mp 2}$

Полином у бројиоцу мора бити мањег степена, зато делимо полиноме.

$(t^6-2t^3):(t^3+t-2) = t^3 - t + \frac{t^2-2t}{t^3+t-2}$

$\underline{-t^6 \pm t^4 \mp 2t^3}$

$-t^4$

$\underline{\mp t^4 \mp t^2 \pm 2t}$

$t^2-2t \rightarrow$ остатак

$$3\int\left(t^3 - t + \frac{t^2-2t}{t^3+t-2}\right)dt = 3\left[\int t^3dt - \int tdt + \int \frac{t^2-2t}{t^3+t-2}dt\right] = 3\left[\frac{t^4}{4} - \frac{t^2}{2} + I_1\right]$$

$I_1 = \int \frac{t^2-2t}{t^3+t-2}dt = \int \frac{t^2-2t}{(t-1)(t^2+t+2)}dt \qquad t^2+t+2 \rightarrow t_{1,2} = \frac{-1\pm\sqrt{1-8}}{2}$

$\frac{t^2-2t}{(t-1)(t^2+t+2)} = \frac{A}{t-1} + \frac{Bt+C}{t^2+t+2} \qquad /(t-1)(t^2+t+2)$

$A = \lim_{t\to 1}\frac{t^2-2t}{t^2+t+2} = \frac{1-2}{1+1+2} = -\frac{1}{4}$

$t^2-2t = A(t^2+t+2) + (Bt+C)(t-1)$

$t^2-2t = At^2 + At + 2A + Bt^2 - Bt + Ct - C$

$t^2-2t = t^2(A+B) + t(A-B+C) + (2A-C)$

$A + B = 1 \longrightarrow B = 1 - A = 1 + \frac{1}{4} = \frac{5}{4}$

$A - B + C = -2$

$2A - C = 0 \rightarrow C = 2A \Longrightarrow C = -\frac{1}{2}$

Провера: $\quad -\frac{1}{4}+\frac{5}{4} = 1 \qquad -\frac{1}{4}-\frac{5}{4}-\frac{1}{2} = -2 \qquad 2\left(-\frac{1}{4}\right) - \left(-\frac{1}{2}\right) = 0$

$\boxed{A = -\frac{1}{4} \quad B = \frac{5}{4} \quad C = -\frac{1}{2}}$ Т

$$I_1 = \int \frac{t^2-2t}{(t-1)(t^2+t+2)}dt = \int \frac{A}{t-1}dt + \int \frac{Bt+C}{t^2+t+2}dt = \int \frac{-\frac{1}{4}}{t-1}dt + \int \frac{\frac{5}{4}t \pm \frac{1}{2}}{t^2+t+2}dt$$

$$\boxed{A = \frac{5}{4}, \qquad B = -\frac{1}{2}, \qquad a = 1, \qquad b = 1, \qquad c = 1}$$

$$I_1 = -\frac{1}{4}\int \frac{1}{t-1}dt + \frac{\frac{5}{4}}{2\cdot 1}\ln|t^2+t+2| + \left(-\frac{1}{2}-\frac{5}{8}\right)\int \frac{1}{t^2+t+2}dt$$

$$I_1 = -\frac{1}{4}\ln|t-1| + \frac{5}{8}\ln|t^2+t+2| - \frac{9}{8}\int \frac{1}{t^2+t+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+2}dt$$

$$\boxed{t+\frac{1}{2} = k, \qquad dt = dk} \to\ -\frac{9}{8}\int \frac{1}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}dt = -\frac{9}{8}\int \frac{1}{k^2+\left(\frac{\sqrt{3}}{2}\right)^2}dt$$

$$-\frac{9}{8}\frac{1}{\frac{\sqrt{3}}{2}}arc\ tg\frac{k}{\frac{\sqrt{3}}{2}} = -\frac{9}{4\sqrt{3}}arc\ tg\frac{2\left(t+\frac{1}{2}\right)}{\sqrt{3}}$$

$$I_1 = -\frac{1}{4}\ln|t-1| + \frac{5}{8}\ln|t^2+t+2| - -\frac{9}{4\sqrt{3}}arc\ tg\frac{2t+1}{\sqrt{3}}$$

$$3\left[\frac{t^4}{4} - \frac{t^2}{2} - \frac{1}{4}\ln|t-1| + \frac{5}{8}\ln|t^2+t+2| - -\frac{9}{4\sqrt{3}}arc\ tg\frac{2t+1}{\sqrt{3}}\right] + C$$

$t = \sqrt[3]{x+2}$ → замена свуда уместо "t"

---

4) $\boxed{\int \frac{x^3-2}{x(x^2+1)}dx}$!!! Пошто су горе и доле полиноми трећег степена тј. истог степена морамо прво поделити полином

$$(x^3-2):(x^3+x) = 1 + \frac{-x-2}{x^3+x} = 1 - \frac{x+2}{x^3+x}$$

$\underline{-x^3 \pm x}$

$-x-2$ → остатак

$$\boxed{\text{Провера Н.З.С. } \frac{x^3+x-x-2}{x^3+x} = \frac{x^3-2}{x^3+x}}$$

$$\int \frac{x^3-2}{x(x^2+1)}dx = \int dx - \int \frac{x+2}{x(x^2+1)}dx = \boxed{x - I_1}$$

$$I_1 = \int \frac{x+2}{x(x^2+1)}dx$$

$$\frac{x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \qquad /x(x^2+1)$$

$$\boxed{A = \lim_{x\to 0}\frac{x+2}{x^2+1} = \frac{2}{1} = 2}$$

$$x+2 = A(x^2+1) + (Bx+C)x$$

$$x+2 = Ax^2 + A + Bx^2 + Cx$$

$$x+2 = x^2(A+B) + xC + A$$

$$A+B = 1, \qquad B = -1, \qquad C = 0, \qquad A = 2$$

$$I_1 = \int \frac{x+2}{x(x^2+1)}dx = \int \frac{A}{x}dx + \int \frac{Bx+C}{x^2+1}dx = \int \frac{2}{x}dx + \int \frac{-1}{x^2+1}dx = 2\ln x - arc\ tg\ x$$

$$I = x - I_1 = x - (2\ln x - arc\ tg\ x)$$

$$\boxed{I = x - 2\ln x + arc\ tg\ x + C}$$

---

5) $\boxed{\int \sqrt{\frac{1-x}{x^4(x+1)}}dx} \quad = \int \frac{\sqrt{1-x}}{\sqrt{x^4}\sqrt{x+1}}dx \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}}$

$$\int \frac{\sqrt{1-x}^2}{x^2\sqrt{x+1}\sqrt{1-x}}dx = \int \frac{1-x}{x^2\sqrt{(x+1)(1-x)}}dx = \int \frac{1-x}{x^2\sqrt{1-x^2}}dx$$

смена: $x = \frac{1}{t}, \qquad dx = -\frac{1}{t^2}dt$

$$\int \frac{1-\frac{1}{t}}{\frac{1}{t^2}\sqrt{1-\frac{1}{t^2}}}\frac{-dt}{t^2} = \int \frac{\frac{t-1}{t}}{\sqrt{\frac{t^2-1}{t^2}}}(-dt) = -\int \frac{\frac{t-1}{t}}{\frac{\sqrt{t^2-1}}{\sqrt{t^2}}}dt = -\int \frac{\frac{t-1}{t}}{\frac{\sqrt{t^2-1}}{t}}dt = -\int \frac{t-1}{\sqrt{t^2-1}}dt$$

$$-\left[\int \frac{t}{\sqrt{t^2-1}}dt - \int \frac{1}{\sqrt{t^2-1}}dt\right] = \left\{t^2-1=k^2, \qquad 2tdt = 2kdk, \qquad dt = \frac{k}{t}dk\right\} =$$

$$= -\int \frac{t}{\sqrt{k}^2}\frac{k}{t}dk + \ln\left|t+\sqrt{t^2-1}\right| = -\int dk + \ln\left|t+\sqrt{t^2-1}\right| = -k + \ln\left|t+\sqrt{t^2-1}\right| =$$

$$= -\sqrt{t^2-1} + \ln\left|t+\sqrt{t^2-1}\right| = -\sqrt{\frac{1}{x^2}-1} + \ln\left|\frac{1}{x}+\sqrt{\frac{1}{x^2}-1}\right| + C$$

---

6) $\boxed{\int e^x \sin 4x\, dx}$

$$\boxed{\begin{array}{ll} u = \sin 4x & dv = e^x dx \\ du = 4\cos 4x\, dx & v = \int e^x dx \\ & v = e^x \end{array}}$$

$I = e^x \sin 4x - 4\int e^x \cos 4x\, dx$

$$\boxed{\begin{array}{ll} u = \cos 4x & dv = e^x dx \\ du = -4\sin 4x\, dx & v = \int e^x dx \\ & v = e^x \end{array}}$$

$$I = e^x \sin 4x - 4\left[e^x \cos 4x + 4\underbrace{\int e^x \sin 4x\, dx}_{I}\right] = e^x \sin 4x - 4e^x \cos 4x - 16I$$

$I + 16I = e^x \sin 4x - e^x \cos 4x$

$17I = e^x(\sin 4x - 4\cos 4x)$

$I = \frac{1}{17}e^x(\sin 4x - 4\cos 4x) + C$

---

7) $\boxed{\int \frac{1}{x\sqrt{x^2+1}}dx}$ $\quad$ $\boxed{I \text{ начин:}}$ $\; x^2+1 = t^2, x^2 = t^2-1, \;\; 2xdx = 2tdt, \qquad dx = \frac{t}{x}dt$

$$\int \frac{1}{x\sqrt{t^2}} \cdot \frac{t}{x}dt = \int \frac{1}{x^2 t}\frac{t}{1}dt = \int \frac{1}{x^2}dt = \int \frac{1}{t^2-1}dt = \frac{1}{2}\ln\frac{t-1}{t+1} = \frac{1}{2}\ln\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1} + C$$

$\boxed{II \text{ начин:}}$ $\; x = \frac{1}{t}, \qquad dx = -\frac{1}{t^2}dt$

$$\int \frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2}+1}}\frac{-dt}{t^2} = -\int \frac{1}{\sqrt{\frac{1+t^2}{t^2}}}dt = -\int \frac{1}{\frac{\sqrt{1+t^2}}{\sqrt{t^2}}}dt = -\int \frac{t}{\sqrt{1+t^2}}dt$$

$1+t^2 = k^2, \qquad 2tdt = 2kdk, \qquad dt = \frac{k}{t}dk$

$$-\int \frac{t}{\sqrt{k^2}}\frac{k}{t}dk = -\int dk = -k = -\sqrt{1+t^2} = -\sqrt{1+\frac{1}{x^2}} + C$$

---

8) $\boxed{\int e^{5x}\cos 6x\,dx}$

$$\boxed{\begin{array}{ll} u = \cos 6x & dv = e^{5x}dx \\ du = -6\sin 6x\,dx & v = \int e^{5x}dx \\ & v = \frac{1}{5}e^{5x} \end{array}}$$

$$I = \frac{1}{5}e^{5x}\cos 6x + \frac{6}{5}\int e^{5x}\sin 6x\,dx$$

$$\boxed{\begin{array}{ll} u = \sin 6x & dv = e^{5x}dx \\ du = 6\cos 6x\,dx & v = \int e^{5x}dx \\ & v = \frac{1}{5}e^{5x} \end{array}}$$

$$I = \frac{1}{5}e^{5x}\cos 6x + \frac{6}{5}\left[\frac{1}{5}e^{5x}\sin 6x - \frac{6}{5}\underbrace{\int e^{5x}\cos 6x\,dx}_{I}\right]$$

$$I = \frac{1}{5}e^{5x}\cos 6x + \frac{6}{25}e^{5x}\sin 6x - \frac{36}{25}I$$

$$I + \frac{36}{25}I = e^{5x}\left[\frac{1}{5}\cos 6x + \frac{6}{5}\cdot\frac{1}{5}\sin 6x\right]$$

$$\frac{61}{25}I = \frac{1}{5}e^{5x}\left(\cos 6x + \frac{6}{5}\sin 6x\right)$$

$$I = \frac{25}{61}\cdot\frac{1}{5}e^{5x}\left(\cos 6x + \frac{6}{5}\sin 6x\right)$$

$$I = \frac{5}{61}e^{5x}\left(\cos 6x + \frac{6}{5}\sin 6x\right) + C$$

---

9) $\boxed{\int arc\sin x\,dx}$

$$\boxed{\begin{array}{ll} u = arc\sin x & dv = dx \\ du = \frac{1}{\sqrt{1-x^2}}dx & v = \int dx \\ & v = x \end{array}}$$

$$I = x\,arc\sin x - \int \frac{x}{\sqrt{1-x^2}}dx$$

$$1 - x^2 = t^2, \qquad -2xdx = 2tdt, \qquad dx = -\frac{t}{x}dt$$

$$I = x\,arc\sin x - \int \frac{x}{\sqrt{t^2}}\cdot\frac{-t}{x}dt = x\,arc\sin x + t = x\,arc\sin x + \sqrt{1-x^2} + C$$

---

10) $\boxed{\int \frac{e^x+1}{e^{2x}-e^x+1}dx}$

$$\boxed{\begin{array}{ll} e^x = t & dx = \frac{dt}{e^x} \\ e^x dx = dt & dx = \frac{dt}{t} \end{array}}$$

$$\int \frac{e^x+1}{e^{2x}-e^x+1}dx = \int \frac{t+1}{t^2-t+1}\frac{dt}{t} = \int \frac{t+1}{t(t^2-t+1)}dx$$

$$\frac{t+1}{t(t^2-t+1)} = \frac{A}{t} + \frac{Bt+C}{t^2-t+1} \qquad /t(t^2-t+1)$$

$$\boxed{A=\lim_{t\to 0}\frac{t+1}{t^2-t+1}=\frac{1}{1}=1}$$

$t+1=A(t^2-t+1)+(Bt+C)t$

$t+1=At^2-At+A+Bt^2+Ct$

$t+1=t^2(A+B)+t(-A+C)+A$

$A+B=0\to \boxed{B=-1}$ $\boxed{A=1}$

$-A+C=1\to \boxed{C=2}$

$$\int\frac{t+1}{t(t^2-t+1)}dx=\int\left(\frac{A}{t}+\frac{Bt+C}{t^2-t+1}\right)dt=\int\frac{1}{t}dt+\int\frac{-t+2}{t^2-t+}dt$$ $\boxed{\begin{matrix}A=-1 & a=1\\ B=2 & b=1\\ & c=1\end{matrix}}$

$$=\ln t+\frac{-1}{2}\ln|t^2-t+1|+\left(2-\frac{(-1)1}{2}\right)\int\frac{1}{t^2-t+1}dt$$

$$=\ln t-\frac{1}{2}\ln|t^2-t+1|+\frac{5}{2}\int\frac{1}{t^2-t+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}dt$$

$$=\ln t-\frac{1}{2}\ln|t^2-t+1|+\frac{5}{2}\int\frac{1}{\left(t-\frac{1}{2}\right)^2-\frac{1}{4}+1}dt$$ $\boxed{t-\frac{1}{2}=k,\qquad dt=dk}$

$$=\ln t-\frac{1}{2}\ln|t^2-t+1|+\frac{5}{2}\int\frac{1}{k^2+\left(\frac{\sqrt{3}}{2}\right)^2}dk$$

$$=\ln t-\frac{1}{2}\ln|t^2-t+1|+\frac{5}{2}\cdot\frac{1}{\frac{\sqrt{3}}{2}}arc\ tg\frac{k}{\frac{\sqrt{3}}{2}}$$

$$=\ln t-\frac{1}{2}\ln|t^2-t+1|+\frac{5}{\sqrt{3}}arc\ tg\frac{2k}{\sqrt{3}}$$

$$=\ln t-\frac{1}{2}\ln|t^2-t+1|+\frac{5}{\sqrt{3}}arc\ tg\frac{2\left(t-\frac{1}{2}\right)}{\sqrt{3}}$$

$$=\ln e^x-\frac{1}{2}\ln|e^{2x}-e^x+1|+\frac{5}{\sqrt{3}}arc\ tg\frac{2e^x-1}{\sqrt{3}}+C$$

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11) $\boxed{\int arc\ tg\sqrt{x}dx}$ $\boxed{\begin{matrix}u=arc\ tg\sqrt{x} & dv=dx\\ du=\frac{1}{\sqrt{x}^2+1}\cdot\frac{1}{2\sqrt{x}}dx & v=\int dx\\ & v=x\end{matrix}}$

$$I=x\ arc\ tg\ \sqrt{x}-\int x\cdot\frac{1}{x+1}\cdot\frac{1}{2\sqrt{x}}dx=x\ arc\ tg\ \sqrt{x}-\frac{1}{2}\int\sqrt{x}\sqrt{x}\cdot\frac{1}{x+1}\cdot\frac{1}{\sqrt{x}}dx$$

$$I=x\ arc\ tg\ \sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}}{x+1}dx$$ $\boxed{x=t^2,\qquad dx=2tdt}$

$$I=x\ arc\ tg\ \sqrt{x}-\frac{1}{2}\int\frac{\sqrt{t^2}}{t^2+1}\cdot 2tdt=x\ arc\ tg\ \sqrt{x}-\frac{2}{2}\int\frac{t^2}{t^2+1}dt$$

$$t^2:(t^2+1)=1-\frac{1}{t^2+1}$$

$$I=x\ arc\ tg\ \sqrt{x}-\int\left(1-\frac{1}{t^2+1}\right)dt=x\ arc\ tg\ \sqrt{x}-\int dt+\int\frac{1}{t^2+1}dt$$

$$I=x\ arc\ tg\ \sqrt{x}-x+arc\ tg\ x+C$$

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12) $\boxed{\int x^2 arc\ tgx\, dx}$

$$\boxed{\begin{array}{ll} u = arc\ tg\ x & dv = x^2dx \\ du = \dfrac{1}{x^2+1}dx & v = \int x^2 dx \\ & v = \dfrac{x^3}{3} \end{array}}$$

$$I = \frac{x^3}{3} arc\ tg\ x - \frac{1}{3}\int \frac{x^3}{x^2+1}dx \qquad x^3 : (x^2+1) = x - \frac{x}{x^2+1}$$

$$I = \frac{x^3}{3} arc\ tg\ x - \frac{1}{3}\int \left(x - \frac{x}{x^2+1}\right)dx$$

$$I = \frac{x^3}{3} arc\ tg\ x - \frac{1}{3}\int x\, dx + \frac{1}{3}\int \frac{x}{x^2+1}dx \qquad \boxed{x^2+1 = t, 2xdx = dt, dx = \frac{dt}{2x}}$$

$$I = \frac{x^3}{3} arc\ tg\ x - \frac{1}{3}\frac{x^2}{2} + \frac{1}{3}\int \frac{x}{t}\frac{dt}{2x} = \frac{x^3}{3} arc\ tg\ x - \frac{x^2}{6} + \frac{1}{6}\int \frac{1}{t}\, dt$$

$$I = \frac{x^3}{3} arc\ tg\ x - \frac{x^2}{6} + \frac{1}{6}\ln t = \frac{x^3}{3} arc\ tg\ x - \frac{x^2}{6} + \frac{1}{6}\ln|x^2+1| + C$$

---

13) $\boxed{\int \frac{e^{3x}}{e^x+2}dx}$

$$\boxed{\begin{array}{ll} e^x = t & dx = \dfrac{dt}{e^x} \\ e^x dx = dt & dx = \dfrac{dt}{t} \end{array}}$$

$$\int \frac{e^{3x}}{e^x+2}dx = \int \frac{t^3}{t+2}\cdot\frac{dt}{t} = \int \frac{t^2}{t+2}dt \qquad t^2 : (t+2) = t - 2 + \frac{4}{t+2}$$

Провера дељења полинома $\Longrightarrow$ Н. З. С.

$$\boxed{t - 2 + \frac{4}{t+2} = \frac{(t-2)(t+2)+4}{t+2} = \frac{t^2-4+4}{t+2} = \frac{t^2}{t+2}}$$

$$\int \frac{t^2}{t+2}dt = \int \left(t - 2 + \frac{4}{t+2}\right)dt = \int tdt - 2\int dt + 4\int \frac{1}{t+2}dt =$$

$$= \frac{t^2}{2} - 2t + 4\ln|t+2| = e^{2x} - 2e^x + 4\ln|e^x+2| + C$$

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14) $\boxed{\int \frac{2}{(2-x)^2}\sqrt[3]{\frac{2-x}{2+x}}dx}$

$$dx = \frac{-6t^2(1+t^3) - (2-2t^3)3t^2}{(1+t^3)^2}dt$$

$$\boxed{\frac{2-x}{2+x} = t^3}$$

$$dx = \frac{t^2[-6-6t^3-6+6t^3]}{(1+t^3)^2}dt$$

$$2 - x = t^3(2+x) \qquad dx = \frac{-12t^2}{(1+t^3)^2}dt$$

$$2 - x = 2t^3 + xt^3$$

$$-x - xt^3 = 2t^3 - 2 \quad /-1$$

$$x + xt^3 = 2 - 2t^3$$

$$x(1+t^3) = 2 - 2t^3$$

$$\boxed{x = \frac{2-2t^3}{1+t^3}}$$

$$\int \frac{2}{\left(2 - \frac{2-2t^3}{1+t^3}\right)^2}\sqrt[3]{t^3}\cdot\frac{-12t^2}{(1+t^3)^2}dt = \int \frac{-24}{\left(\frac{2+2t^3-2+2t^3}{1+t^3}\right)^2}\cdot\frac{t^3}{(1+t^3)^2}dt =$$

$$\int -\frac{24}{\frac{(4t^3)^2}{(1+t^3)^2}}\frac{t^3}{(1+t^3)^2}dt = -\frac{24}{16}\int \frac{1}{t^6}t^3 dt = -\frac{3}{2}\int \frac{1}{t^3}dt = -\frac{3}{2}\cdot\left(-\frac{1}{2t^2}\right) + \frac{3}{4}\cdot\frac{\frac{1}{1}}{\left(\sqrt[3]{\frac{2-x}{2+x}}\right)^2} =$$

$$= \frac{3}{4}\cdot\sqrt[3]{\frac{2-x}{2+x}}^2 + C$$

---

15) $\boxed{\int \frac{21x^2-80x+48}{x^3-6x^2+8x}dx}$

$x(x^2-6x+8)$

$x_{1,2} = \frac{6 \pm \sqrt{36-22}}{2}$ $\quad x_1 = 4, \qquad x_2 = 2$

$$\frac{21x^2-80x+48}{x(x-4)(x-2)} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x-2}$$

$$A = \lim_{x\to 0}\frac{21x^2-80x+48}{(x-4)(x-2)} = \frac{48}{8} = 6$$

$$B = \lim_{x\to 4}\frac{21x^2-80x+48}{x(x-2)} = \frac{21\cdot 16 - 80\cdot 4 + 48}{4\cdot 2} = 8$$

$$C = \lim_{x\to 2}\frac{21x^2-80x+48}{x(x-4)} = \frac{21\cdot 4 - 80\cdot 2 + 48}{2\cdot(-2)} = 7$$

$$\int \frac{21x^2-80x+48}{x^3-6x^2+8x}dx = \int\left(\frac{A}{x} + \frac{B}{x-4} + \frac{C}{x-2}\right)dx = 6\int\frac{1}{x}dx + 8\int\frac{1}{x-4}dx + 7\int\frac{1}{x-2}dx$$

$$= 6\ln x + 8\ln|x-4| + 7\ln|x-2| = 6\ln x + 8\ln|x-4| + 7\ln|x-2| + C$$

---

16) $\boxed{\int \frac{2x^2+3x-4}{x^3-2x^2+x-2}dx}$

I начин

$$\begin{aligned} x^3-2x^2+x-2 &= x^3+x-2x^2-2 \\ &= x(x^2+1)-2(x^2+1) \\ &= (x^2+1)(x-2) \end{aligned}$$

$x^3-2x^2+x-2 = 0$

0	$-2 \neq 0$
1	$1-2+1-2 \neq 0$
-1	$-1-2-1-2 \neq 0$
2	$8-8+2-2 = 0$

$(x^3-2x^2+x-2):(x-2) = x^2+1$

$x^3-2x^2+x-2 = (x^2+1)(x-2)$

$$\int\frac{2x^2+3x-4}{x^3-2x^2+x-2}dx = \int\frac{2x^2+3x-4}{(x-2)(x^2+1)}dx$$

$$\frac{2x^2+3x-4}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1} \qquad /(x-2)(x^2+1)$$

$$\boxed{A = \lim_{x\to 2}\frac{2x^2+3x-4}{x^2+1} = \frac{2\cdot 4+3\cdot 2-4}{4+1} = \frac{10}{5} = \boxed{2}}$$

$2x^2+3x-4 = A(x^2+1)+(Bx+C)(x-2)$

$2x^2+3x-4 = Ax^2+A+Bx^2-2Bx+Cx-2C$

$2x^2+3x-4 = x^2(A+B)+x(-2B+C)+(A-2C)$

$$\begin{aligned} A+B &= 2 \\ -2B+C &= 3 \\ A-2C &= -4 \end{aligned} \qquad \begin{aligned} B &= 0 \\ C &= 3 \end{aligned} \quad A = 2$$

$$\int\frac{2x^2+3x-4}{(x-2)(x^2+1)}dx = \int\left(\frac{A}{x-2}+\frac{Bx+C}{x^2+1}\right)dx = \int\frac{2}{x-2}dx + \int\frac{3}{x^2+1}dx =$$

$= 2\ln|x-2| + 3arc\ tg\ x + C$

---

17) $\boxed{\int \frac{1}{1+\sqrt[3]{x+1}}dx}$ $\boxed{\begin{matrix} x+1=t^3 \\ dx = 3t^2dt \end{matrix}}$

$\int \frac{1}{1+\sqrt[3]{t^3}} \cdot 3t^2dt = 3\int \frac{t^2}{t+1}dt$ Делимо полином јер је горњи већег степена, а треба обратно

$t^2 : (t+1) = t - 1 + \frac{1}{t+1}$

$3\int \frac{t^2}{t+1}dt = 3\int \left(t - 1 + \frac{1}{t+1}\right)dt = 3\left[\int tdt - \int dt + \int \frac{1}{t+1}dt\right] =$

$= 3\left[\frac{t^2}{2} - t + \ln|t+1|\right] = 3\left[\frac{\sqrt[3]{x+1}^2}{2} - \sqrt[3]{x+1} + \ln\left|\sqrt[3]{x+1}+1\right|\right] + C$

---

18) $\boxed{\int \sin x\, e^x dx}$ $\boxed{\begin{matrix} u = \sin x & dv = e^x dx \\ du = \cos x\, dx & v = \int e^x dx \\ & v = e^x \end{matrix}}$

$I = e^x \sin x - \int e^x \cos x\, dx$ $\boxed{\begin{matrix} u = \cos x & dv = e^x dx \\ du = -\sin x\, dx & v = \int e^x dx \\ & v = e^x \end{matrix}}$

$I = e^x \sin x - \left[e^x \cos x + \underbrace{\int e^x \sin x\, dx}_{I}\right]$

$I = e^x \sin x - e^x \cos x - I$

$2I = e^x(\sin x - \cos x)$

$I = \frac{e^x}{2}(\sin x - \cos x) + C$

---

19) $\boxed{\int \frac{(x+1)^3}{x^2+2x-3}dx}$

$(x^3 + 3x^2 + 3x + 1) : (x^2 + 2x - 3) = x + 1 + \frac{4x+4}{x^2+2x-3}$

$\underline{-x^3 \pm 2x^2 \mp 3x}$

$x^2 + 6x + 1$

$\underline{-x^2 \pm 2x \mp 3}$

$4x + 4$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(x+1)^3 = x^3 + 3x^2 + 3x + 1$

$\int \frac{(x+1)^3}{x^2+2x-3}dx = \int \left(x + 1 + \frac{4x+4}{x^2+2x-3}\right)dx = \int xdx + \int dx + 4\int \frac{x+1}{x^2+2x-3}dx$

$A = 1, \quad B = 1, \quad a = 1, \quad b = 2, \quad c = -3$

$= \frac{x^2}{2} + x + 4\left(\frac{1}{2}\ln|x^2+2x-3| + \left(1 - \frac{1}{2}\right)\int \frac{1}{x^2+2x-3}dx\right)$

$= \frac{x^2}{2} + x + 2\ln|x^2+2x-3| + 2\int \frac{1}{x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 3}dx$

$$= \frac{x^2}{2} + x + 2\ln|x^2+2x-3| + 2\int \frac{1}{(x+1)^2-4}dx$$

$x+1=t, \qquad dx=dt$

$$= \frac{x^2}{2} + x + 2\ln|x^2+2x-3| + 2\int \frac{1}{t^2-2^2}dt$$

$$= \frac{x^2}{2} + x + 2\ln|x^2+2x-3| + 2\cdot\frac{1}{2\cdot 2}\int \frac{t-2}{t+2}dt$$

$$= \frac{x^2}{2} + x + 2\ln|x^2+2x-3| + 2\cdot\frac{1}{2\cdot 2}\ln\frac{t-2}{t+2}$$

$$= \frac{x^2}{2} + x + 2\ln|x^2+2x-3| + \frac{1}{2}\ln\frac{x+1-2}{x+1+2}$$

$$= \frac{x^2}{2} + x + 2\ln|x^2+2x-3| + \frac{1}{2}\ln\frac{x-1}{x+3} + C$$

---

20) $\boxed{\int arc\sin^2 x\,dx}$

$$\boxed{\begin{matrix} u = arc\sin^2 x & dv = dx \\ du = 2\,arc\sin x\cdot\frac{1}{\sqrt{1-x^2}}dx & v=\int dx \\ & v = x\end{matrix}}$$

$$\int arc\sin^2 x\,dx = x\,arc\sin^2 x - 2\int arc\sin x\cdot\frac{1}{\sqrt{1-x^2}}dx$$

$$\boxed{\begin{matrix} u = arc\sin x & dv = \frac{x}{\sqrt{1-x^2}}dx & 1-x^2=t^2 \\ du=\frac{1}{\sqrt{1-x^2}}dx & v=\int\frac{x}{\sqrt{1-x^2}}dx & -2xdx = 2tdt \\ & & dx = -\frac{t}{x}dt\end{matrix}}$$

$$\boxed{\begin{matrix} v=\int\frac{x}{\sqrt{t^2}}\cdot-\frac{t}{x}dt \\ v=-\int dt \\ v=-t \\ v=-\sqrt{1-x^2}\end{matrix}}$$

$$= x\,arc\sin^2 x - 2\left[-\sqrt{1-x^2}\cdot arc\sin x + \int\frac{1}{\sqrt{1-x^2}}\cdot\sqrt{1-x^2}dx\right]$$

$$= x^2\,arc\sin^2 x + 2\sqrt{1-x^2}arc\sin x + \int dx$$

$$= x^2\,arc\sin^2 x + 2\sqrt{1-x^2}arc\sin x + x + C$$

---

21) $\boxed{\int \sin\sqrt{x}\,dx}$ $\qquad \boxed{\begin{matrix} x=t^2 \\ dx = 2tdt\end{matrix}}$

$2\int t\sin t\,dt$ $\qquad \boxed{\begin{matrix} & dv=\sin t\,dt \\ u=t & v=\int\sin t\,dt \\ du=dt & \\ & v=-\cos t\end{matrix}}$

$$2\int t\sin t\,dt = 2\left[-t\cos t + \int\cos t\,dt\right] = -2t\cos t + \sin t = -2\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x} + C$$

---

22) $\boxed{\int\frac{x+\sqrt{x}+\sqrt[3]{x^2}}{x\left(1+\sqrt[3]{x}\right)}dx} = \int\frac{x+\sqrt{x}+\sqrt[3]{x}^2}{x\left(1+\sqrt[3]{x}\right)}dx$ $\qquad \boxed{\begin{matrix} x=t^6 \\ dx=6t^5dt\end{matrix}}$

$$= \int\frac{t^6+\sqrt{t^6}+\sqrt[3]{t^6}^2}{t^6\left(1+\sqrt[3]{t^6}\right)}\cdot 6t^5dt = 6\int\frac{t^6+t^3+t^4}{t^2+1}dt$$

Полином у бројиоцу је већег степена него у имениоцу, па морамо делити полиноме

$$(t^6+t^4+t^3):(t^2+1)=t^4+t-\frac{t}{t^2+1}$$

$$\underline{-t^6 \pm t^4}$$

$$t^3$$

$$\underline{-t^3 \pm t}$$

$$-t$$

Провера $\Longrightarrow$ Н.З.С. $\dfrac{t^6+t^4+t^3+t-t}{t^2+1}=\dfrac{t^6+t^4+t^3}{t^2+1}$

$$6\int\frac{t^6+t^3+t^4}{t^2+1}dt=6\int\left(t^4+t-\frac{t}{t^2+1}\right)dt=\left\{t^2+1=k, 2tdt=dk, dt=\frac{dk}{2t}\right\}$$

$$=6\left[\frac{t^5}{5}+\frac{t^2}{2}-\int\frac{t}{k}\frac{dt}{2t}\right]=6\left[\frac{t^5}{5}+\frac{t^2}{2}-\ln k\right]=6\left[\frac{t^5}{5}+\frac{t^2}{2}-\ln|t^2+1|\right]=$$

$$=6\left[\frac{\sqrt[6]{t}^5}{5}+\frac{\sqrt[6]{t}^2}{2}-\ln\left|\sqrt[6]{t}^2+1\right|\right]+C$$

---

23) $\displaystyle\int\sin 2x\, e^{3x}dx$

$$u=\sin 2x \quad du= \quad dx \quad du=2\cos 2x\,dx \qquad dv=e^{3x}dx \quad v=\int e^{3x}dx \quad v=\frac{1}{3}e^{3x}$$

$$I=\frac{1}{3}e^{3x}\sin 2x-\frac{2}{3}\int\cos 2x\, e^{3x}dx \qquad u=\cos 2x \quad du=-2\sin 2x\,dx \qquad dv=e^{3x}dx \quad v=\int e^{3x}dx \quad v=\frac{1}{3}e^{3x}$$

$$I=\frac{1}{3}e^{3x}\sin 2x-\frac{2}{3}\left[\frac{1}{3}e^{3x}\cos 2x+\frac{2}{3}\underbrace{\int\sin 2x\, e^{3x}dx}_{I}\right]$$

$$I=\frac{1}{3}e^{3x}\sin 2x-\frac{2}{9}e^{3x}\cos 2x-\frac{4}{9}I$$

$$I+\frac{4}{9}I=\frac{1}{3}e^{3x}\left(\sin 2x-\frac{2}{3}\cos 2x\right)$$

$$\frac{13}{9}I=\frac{1}{3}e^{3x}\left(\sin 2x-\frac{2}{3}\cos 2x\right)$$

$$I=\frac{9}{13}\cdot\frac{1}{3}e^{3x}\left(\sin 2x-\frac{2}{3}\cos 2x\right)$$

$$I=\frac{3}{13}e^{3x}\left(\sin 2x-\frac{2}{3}\cos 2x\right)+C$$

---

24) $\displaystyle\int\frac{1}{1+\sin x+\cos x}dx$

$$t=tg\frac{x}{2} \quad \sin x=\frac{2t}{t^2+1} \quad dx=\frac{2dt}{t^2+1} \quad \cos x=\frac{1-t^2}{t^2+1}$$

$$\int\frac{1}{1+\frac{2t}{t^2+1}+\frac{1-t^2}{t^2+1}}\cdot\frac{2dt}{t^2+1}=\int\frac{1}{\frac{t^2+1+2t+1-t^2}{t^2+1}}\cdot\frac{2dt}{t^2+1}=\frac{2}{2}\int\frac{1}{t+1}dt=$$

$= \ln|t+1| = \ln\left|tg\frac{x}{2}+1\right| + C$

---

25) $\boxed{\int \frac{x^3+1}{x^3-5x^2+6x}dx}$ $\rightarrow$ $x(x^2-5x+6) = x(x-3\_(x-2)$ $\quad x_{1,2} = \frac{5 \pm \sqrt{25-24}}{2} \qquad x_1 = 3, \qquad x_2 = 2$

$(x^3+1):(x^3-5x^2+6x) = 1 + \frac{5x^2-6x+1}{x^3-5x^2+6x}$

$\int \frac{x^3+1}{x^3-5x^2+6x}dx = \int\left(1+\frac{5x^2-6x+1}{x^3-5x^2+6x}\right)dx = \int dx + \int \frac{5x^2-6x+1}{x(x-3)(x-2)}dx = \boxed{x + I_1}$

$\frac{5x^2-6x+1}{x(x-3)(x-2)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x-2}$

$A = \lim_{x\to 0}\frac{5x^2-6x+1}{(x-3)(x-2)} = \frac{1}{(-3)(-2)} = \frac{1}{6}$

$B = \lim_{x\to 3}\frac{5x^2-6x+1}{x(x-2)} = \frac{5\cdot 9 - 6\cdot 3+1}{3\cdot(1)} = \frac{28}{3}$

$C = \lim_{x\to 2}\frac{5x^2-6x+1}{x(x-3)} = \frac{5\cdot 4 - 6\cdot 2+1}{2\cdot(-1)} = -\frac{9}{2}$

$\int \frac{5x^2-6x+1}{x(x-3)(x-2)}dx = \int\left(\frac{A}{x}+\frac{B}{x-3}+\frac{C}{x-2}\right)dx = \frac{1}{6}\int\frac{1}{x}dx + \frac{28}{3}\int\frac{1}{x-3}dx - \frac{9}{2}\int\frac{1}{x-2}dx$

$= \frac{1}{6}\ln x + \frac{28}{3}\ln|x-3| - \frac{9}{2}\ln|x-2| + C$

---

26) $\boxed{\int \frac{1}{x^4+x^3+x^2}dx}$ Пошто ништа од A, B, C, D не можемо наћи преко лимеса приморани смо да радимо једначине 4x4 што ни мало није лако или … нова идеја

$\frac{1}{x^2(x^2+x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{Cx+D}{x^2+x+1} \qquad x^2+x+1 = 0 \rightarrow x_{1,2} = \frac{-1\pm\sqrt{1-4}}{2}$

$\frac{1}{x^2(x^2+x+1)} = \frac{x^2+x+1 \quad -x^2-x}{x^2(x^2+x+1)} = \frac{x^2+x+1}{x^2(x^2+x+1)} - \frac{x^2+x}{x^2(x^2+x+1)} =$

$= \frac{1}{x^2} - \frac{x(x+1)}{x^2(x^2+x+1)} = \frac{1}{x^2} - \frac{x+1}{x^2+x+1}$

$\int \frac{1}{x^4+x^3+x^2}dx = \int\frac{1}{x^2(x^2+x+1)}dx = \int\left(\frac{1}{x^2} - \frac{x+1}{x^2+x+1}\right)dx =$

$= -\frac{1}{x} - \left[\frac{1}{2}\ln|x^2+x+1| + \left(-\frac{1}{2}\right)\int\frac{1}{x^2+x+1}dx\right]$ $\quad \boxed{A = 1 \;\; B = 1 \;\; a = 1 \;\; b = 1 \;\; c = 1}$

$= -\frac{1}{x} - \frac{1}{2}\ln|x^2+x+1| + \frac{1}{2}\int\frac{1}{x^2+x+\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}dx$

$= -\frac{1}{x} - \frac{1}{2}\ln|x^2+x+1| + \frac{1}{2}\int\frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}dx$ $\quad \boxed{x + \frac{1}{2} = t, \qquad dx = dt}$

$= -\frac{1}{x} - \frac{1}{2}\ln|x^2+x+1| + \frac{1}{2}\int\frac{1}{t^2+\sqrt{\frac{3}{4}}^2}dt = -\frac{1}{x} - \frac{1}{2}\ln|x^2+x+1| + \frac{1}{2}\int\frac{1}{t^2+\left(\frac{\sqrt{3}}{2}\right)^2}dt$

$$= -\frac{1}{x} - \frac{1}{2}\ln|x^2+x+1| + \frac{1}{2}\cdot\frac{1}{\frac{\sqrt{3}}{2}} arc\ tg \frac{t}{\frac{\sqrt{3}}{2}} = -\frac{1}{x} - \frac{1}{2}\ln|x^2+x+1| + \frac{1}{\sqrt{3}} arc\ tg \frac{2t}{\sqrt{3}} =$$

$$= -\frac{1}{x} - \frac{1}{2}\ln|x^2+x+1| + \frac{1}{\sqrt{3}} arc\ tg \frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}} = -\frac{1}{x} - \frac{1}{2}\ln|x^2+x+1| + \frac{1}{\sqrt{3}} arc\ tg \frac{2x+1}{\sqrt{3}} + C$$

---

27) $\boxed{\int x \cdot \ln\frac{x+1}{x-1}\,dx}$

$$\boxed{\begin{array}{ll} u = \ln\frac{x+1}{x-1} & dv = x\,dx \\ du = \frac{1}{\frac{x+1}{x-1}}\cdot\frac{1(x-1)-(x+1)\cdot 1}{(x-1)^2}dx & v = \int x\,dx \\ du = -\frac{2}{x^2-1}dx & v = \frac{x^2}{2} \end{array}}$$

$$\int x\cdot\ln\frac{x+1}{x-1}dx = \frac{x^2}{2}\ln\frac{x+1}{x-1} - \int\frac{x^2}{2}\cdot\frac{-2}{x^2-1}dx = \frac{x^2}{2}\ln\frac{x+1}{x-1} + \int\frac{x^2}{x^2-1}dx$$

$$x^2 : (x^2-1) = 1 + \frac{1}{x^2-1}$$

$$= \frac{x^2}{2}\ln\frac{x+1}{x-1} + \int\left(1+\frac{1}{x^2-1}\right)dx = \frac{x^2}{2}\ln\frac{x+1}{x-1} + \int dx + \int\frac{1}{x^2-1}dx =$$

$$= \frac{x^2}{2}\ln\frac{x+1}{x-1} + x + \frac{1}{2}\ln\frac{x-1}{x+1} + C$$

---

28) $\boxed{\int e^{3x}\sin 4x\,dx}$

$$\boxed{\begin{array}{ll} & dv = e^{3x}dx \\ u = \sin 4x & v = \int e^{3x}dx \\ du = 4\cos 4x\,dx & v = \frac{1}{3}e^{3x} \end{array}}$$

$$\int e^{3x}\sin 4x\,dx = \frac{1}{3}e^{3x}\sin 4x - \frac{4}{3}\int e^{3x}\cos 4x\,dx$$

$$\boxed{\begin{array}{ll} & dv = e^{3x}dx \\ u = \cos 4x & v = \int e^{3x}dx \\ du = -4\sin 4x\,dx & v = \frac{1}{3}e^{3x} \end{array}}$$

$$I = \frac{1}{3}e^{3x}\sin 4x - \frac{4}{3}\left[\frac{1}{3}e^{3x}\cos 4x + \frac{4}{3}\underbrace{\int e^{3x}\sin 4x\,dx}_{I}\right]$$

$$I = \frac{1}{3}e^{3x}\sin 4x - \frac{4}{3}\cdot\frac{1}{3}e^{3x}\cos 4x - \frac{16}{9}I$$

$$I + \frac{16}{9}I = \frac{1}{3}e^{3x}\left(\sin 4x - \frac{4}{3}\cos 4x\right)$$

$$\frac{25}{9}I = \frac{1}{3}e^{3x}\left(\sin 4x - \frac{4}{3}\cos 4x\right)$$

$$I = \frac{9}{25}\cdot\frac{1}{3}e^{3x}\left(\sin 4x - \frac{4}{3}\cos 4x\right)$$

$$I = \frac{3}{25}e^{3x}\left(\sin 4x - \frac{4}{3}\cos 4x\right)$$

---

29) $\boxed{\int x\,arc\,tg\,x\,dx}$

$$\boxed{\begin{matrix} u = arc\,tg\,x & dv = xdx \\ du = \frac{1}{x^2+1}dx & v = \int x\,dx \\ & v = \frac{x^2}{2} \end{matrix}}$$

$$\int x\,arc\,tg\,x\,dx = \frac{x^2}{2}arc\,tg\,x - \int \frac{x^2}{2}\cdot\frac{1}{x^2+1}dx = \frac{x^2}{2}arc\,tg\,x - \frac{1}{2}\int \frac{x^2}{x^2+1}dx$$

$$x^2:(x^2+1) = 1 - \frac{1}{x^2+1}$$

$$= \frac{x^2}{2}arc\,tg\,x - \frac{1}{2}\int\left(1-\frac{1}{x^2+1}\right)dx = \frac{x^2}{2}arc\,tg\,x - \frac{1}{2}(x - arc\,tg\,x) + C$$

---

30) $\boxed{\int \frac{2x\,arc\,tg\,x^2}{x^4+1}\,dx}$

$$\boxed{\begin{matrix} x^2 = t \\ 2xdx = dt \\ dx = \frac{dt}{2x} \end{matrix}} \quad \boxed{\begin{matrix} arc\,tg\,t = k \\ \frac{1}{t^2+1}dt = dk \\ dt = (t^2+1)dk \end{matrix}}$$

$$\int \frac{2x\,arc\,tg\,x^2}{x^4+1}\,dx = \int \frac{2x\,arc\,tg\,t}{t^2+1}\cdot\frac{dt}{2x} = \int \frac{arc\,tg\,t}{t^2+1}dt = \int \frac{k}{t^2+1}(t^2+1)dk = \int kdk = \frac{k^2}{2} =$$

$$= \frac{(arc\,tg\,t)^2}{2} = \frac{(arc\,tg\,x^2)^2}{2} + C$$

---

31) $\boxed{\int \frac{1}{x^5-x^2}\,dx}$

$$x^5 - x^2 = x^2(x^3-1)$$
$$= x^2(x-1)(x^2+x+1)$$
$$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\frac{1}{x^2(x-1)(x^2+x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1} + \frac{Dx+E}{x^2+x+1}$$

$\boxed{5x5}$ → врло сложено ⟹ постоји фора

$$\frac{1}{x^5-x^2} = \frac{1}{x^2(x^3-1)} = -\frac{-1}{x^2(x^3-1)} = -\frac{x^3-1 \quad -x^3}{x^2(x^3-1)} = -\left(\frac{x^3-1}{x^2(x^3-1)} - \frac{x^3}{x^2(x^3-1)}\right) =$$

$$= -\left(\frac{1}{x^2} - \frac{x}{x^3-1}\right) = -\frac{1}{x^2} + \frac{x}{x^3-1}$$

$$\int \frac{1}{x^5-x^2}\,dx = \int\left(-\frac{1}{x^2}+\frac{x}{x^3-1}\right)dx = \int -\frac{1}{x^2}dx + \int \frac{x}{x^3-1}dx = +\frac{1}{x} + I_1$$

$$I_1 = \int \frac{x}{x^3-1}dx$$

$$\frac{x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \qquad /(x-1)(x^2+x+1)$$

$$A = \lim_{x\to1}\frac{x}{x^2+x+1} = \frac{1}{1+1+1} = \frac{1}{3}$$

$$x = A(x^2+x+1) + (Bx+C)(x-1)$$
$$x = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$$
$$x = x^2(A+B) + x(A-B+C) + (A-C)$$

$$A + B = 0 \quad \to B = -\frac{1}{3}$$

$$A - B + C = 1 \qquad\qquad A = \frac{1}{3}$$

$$A - C = 0 \quad \to C = \frac{1}{3}$$

$$\int \frac{x}{x^3-1}dx = \int \left(\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}\right)dx = \int \frac{\frac{1}{3}}{x-1}dx + \int \frac{-\frac{1}{3}x+\frac{1}{3}}{x^2+x+1}dx =$$

$$= \frac{1}{3}\int \frac{1}{x-1}dx + \frac{1}{3}\int \frac{-x+1}{x^2+x+1}dx \qquad \begin{matrix} A=-1 \\ B=1 \end{matrix} \quad \begin{matrix} a=1 \\ a=1 \\ a=1 \end{matrix}$$

$$= \frac{1}{3}\ln|x-1| + \frac{1}{3}\left[-\frac{1}{2}\ln|x^2+x+1| + \left(1-\frac{-1}{2}\right)\int \frac{1}{x^2+x+1}dx\right]$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| + \frac{1}{3}\cdot\frac{3}{2}\int \frac{1}{x^2+x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| + \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}dx \qquad \boxed{\begin{matrix} x+\frac{1}{2}=t \\ dx=dt \end{matrix}}$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| + \frac{1}{2}\int \frac{1}{t^2+\left(\frac{\sqrt{3}}{2}\right)^2}dt$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| + \frac{1}{2}\cdot\frac{1}{\frac{\sqrt{3}}{2}}arc\ tg\frac{t}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| + \frac{1}{\sqrt{3}}arc\ tg\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}}$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| + \frac{1}{\sqrt{3}}arc\ tg\frac{2x+1}{\sqrt{3}} + C$$

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32) $\boxed{\int e^{x+e^x}dx} = \int e^x\cdot e^{e^x}dx \qquad \boxed{\begin{matrix} e^x=t & dx=\frac{dt}{e^x} \\ e^x dx=dt & dx=\frac{dt}{t} \end{matrix}}$

$$\int t\cdot e^t\frac{dt}{t} = \int e^t dt = e^t = e^{e^x}+C$$

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33) $\boxed{\int \ln\frac{x+1}{x-1}dx}$

$$\boxed{\begin{matrix} u=\ln\frac{x+1}{x-1} & \\ du=\frac{1}{\frac{x+1}{x-1}}\cdot\frac{1(x-1)-(x+1)\cdot 1}{(x-1)^2}dx \quad dx & \begin{matrix} dv=dx \\ v=\int dx \\ v=x \end{matrix} \\ du=-\frac{2}{x^2-1} & \end{matrix}}$$

$$\int \ln\frac{x+1}{x-1}dx = x\ln\frac{x+1}{x-1} - \int\frac{-2}{x^2-1}x\,dx = x\ln\frac{x+1}{x-1} + 2\int\frac{x}{x^2-1}dx$$

$$x^2-1=t, \qquad 2xdx=dt, \qquad dx=\frac{dt}{2x}$$

$$= x\ln\frac{x+1}{x-1} + 2\int\frac{x}{t}\cdot\frac{dt}{2x} = x\ln\frac{x+1}{x-1} + \ln t = x\ln\frac{x+1}{x-1} + \ln|x^2-1| + C$$

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34) $\boxed{\int x^3 e^{x^2} dx}$

$$\boxed{\begin{array}{c} x^2 = t \\ 2x dx = dt \\ dx = \dfrac{dt}{2x} \end{array}}$$

$$\int x^3 e^{x^2} dx = \int x^3 e^t \frac{dt}{2x} = \frac{1}{2}\int x^2 e^t dt = \frac{1}{2}\int t\, e^t dt \qquad \boxed{\begin{array}{cc} & dv = e^t dt \\ u = t & \\ du = dt & v = \int e^t dt \\ & v = e^t \end{array}}$$

$$= \frac{1}{2}\left[t\, e^t - \int e^t dt\right] = \frac{1}{2}[t\, e^t - e^t] = \frac{1}{2}e^t(t-1) = \frac{1}{2}e^{x^2}(x^2-1) + C$$
